Displacement of equilibria and generation of n-double wing attractors in the piecewise linearized Lorenz system

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Abstract. In this paper, a method for generating double wing attractors in unstable dissipative systems (UDS) is presented. The example at hand is the Lorenz system. It is demonstrated that by replacing one of the constant parameters in the system by a suitably designed piecewise linear function, it is possible to generate new equilibria, which ultimately results in the onset of n-double wing attractors. The performance of the proposed method is illustrated by numerical simulations

Introduction

The Unstable Dissipative Systems (UDS) are described by third order piecewise linear systems with unstable hyperbolic focus-saddle equilibria [1, 2]. These systems may exhibit chaotic scroll or wing behavior if at least two equilibria of the same type are located in specific locations in the state space. However, if more equilibria with the same characteristics are added to the system, the attractors will displace toward the recently added equilibria [3].

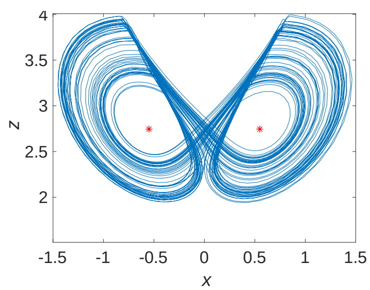
Problem statement

Consider a linearized version of the Lorenz system given by: $\dot{x} = y - x$; $\dot{y} = -z \cdot sign(x) + c \cdot sign(x) - a \cdot y$; $\dot{z} = x \cdot sign(y) - b \cdot z$, where $a, b,$ and c are positive parameters and $sign(\cdot)$ is the sign function. This system satisfies the UDS characteristics [4] and furthermore, the system exhibits a double wing attractor, as shown in Figure 1 a). The question that emerges is, does this double wing behavior can be 'extended' into the z-axis such that n-double scroll attractors emerge?

Proposed method and results

Here, we introduce a modified version of the equilibria displacement technique presented in [3]. In particular, the parameter c in the Lorenz system described above is converted into a 'switching' parameter, i.e., it is replaced by a piecewise linear function such that new equilibria and switching surfaces in the z-axis are introduced. The values of the switching parameter c, as well as the location of the switching surfaces, are obtained by using an exponential function and linear regression. In this way, it is possible to generate n-double wing attractors in the linearized Lorenz system. As an example, Figure 1b) shows a 7-double wing attractor,

a)



b)

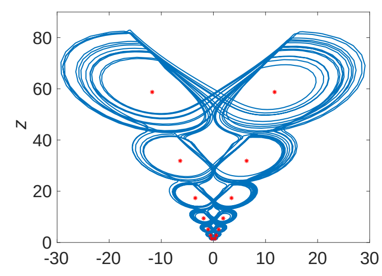


Figure 1: a) Projection onto the (x, z) plane for the linearized Lorenz system given in [4]. b) Projection of a designed 7-double wing attractor.

Conclusions

A method for generating n-double wing attractors in the linearized Lorenz system is presented. The method may be applicable to other UDS systems. A potential application of the results presented here is in cryptography.

References

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