

# Dynamic sensitivity analysis of transient responses for nonlinear structures

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**Abstract.** A direct sensitivity analysis method for nonlinear dynamic responses is presented in this paper. By directly differentiating the nonlinear equation of motions, the dynamic response and corresponding sensitivity can be synchronously determined using a forward time integration algorithm. The proposed method is applied to perform sensitivity analysis for the Duffing oscillator. Results show that the proposed method is successfully applied to the sensitivity computation of periodic and quasi-periodic nonlinear vibrations.

## Introduction

Structural nonlinearities are widespread in engineering and should not be neglected in modern engineering design. Dynamic performance of the nonlinear system can be assessed by investigating how structural parameters affect the nonlinear dynamic responses, and sensitivity analysis of nonlinear dynamic responses is the critical issue. The sensitivities of the responses for linear systems can be calculated by using three methods [1]: the finite difference method (FDM), the adjoint variable method (AVM), and the direct differentiation method (DDM). The DDM is an effective choice for forward time integration and can be successfully applied to sensitivity analysis for the differentiable equations. Conte et al. [2] and Gu et al. [3] proposed the DDM-based response sensitivity analysis approach to structure with plasticity material under dynamic loading conditions. Scott and Azad [4] applied the DDM to a force-based element formulation with material and geometric nonlinearity to compute the response sensitivities. Few attempts have been conducted on the problem of dynamic response sensitivity analysis for an assembled structure, which contains smooth nonlinearities in local. The DDM also can be applied to non-smooth dynamical systems after some smoothing approaches are conducted to the non-smooth restoring forces. In this paper, a direct sensitivity analysis method for nonlinear dynamic responses is proposed.

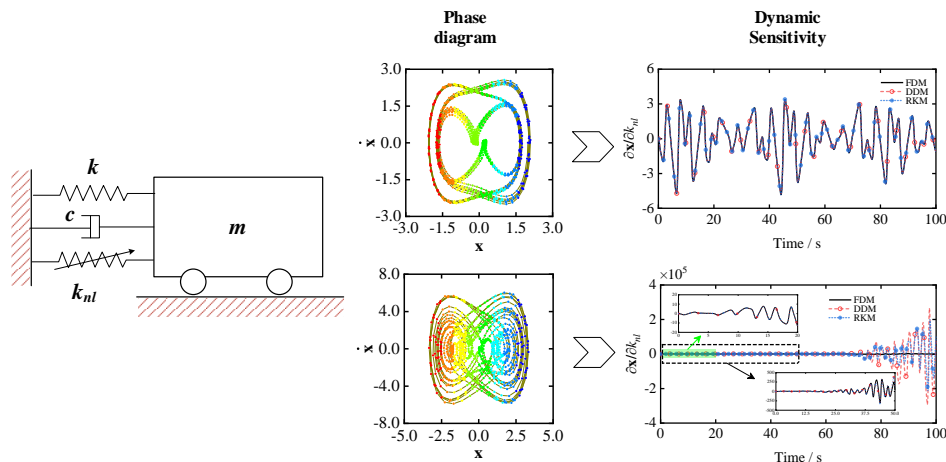


Figure 1: Dynamic responses and corresponding sensitivities for the Duffing oscillator.

## Results and discussion

The method is successfully applied for sensitivity analysis of dynamic responses for nonlinear systems by comparing the FDM. The behaviours of the nonlinear dynamic response sensitivities are investigated by different dynamical vibrations, e.g., quasi-periodic vibration, and chaotic vibration. Results show that the dynamic sensitivity of the quasi-periodic dynamic response remains bounded and settles into a regular region. When the structure vibrates in a chaotic region, the proposed method gives more consistent solution than that obtained by the finite difference method.

## References

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