Bifurcation analysis of a Duffing oscillator with a multi-segmented freeplay nonlinearity

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Abstract. Freeplay is a common type of nonlinearity in dynamical systems, and it can cause discontinuity-induced bifurcations that may bring about damaging responses in a system. In this work, numerical bifurcation analysis is performed on a forced Duffing oscillator with a multi-segmented freeplay nonlinearity. A time integration method developed in Matlab® with Event Location is used. The focus of the investigation is on grazing and grazing/sliding bifurcations and how they change the system physics. Preliminary results indicate grazing bifurcations are highly dependent on the symmetry and stiffness of the freeplay nonlinearity and can increase the number of possible solutions at a given forcing frequency.

Introduction

Freeplay nonlinearities are commonly present in mechanical systems such as aircraft control parts or gear trains. Freeplay can induce grazing and grazing/sliding bifurcations, where the system motion drops to zero velocity when it contacts a nonlinearity boundary for a moment or for a short time period, respectively. These discontinuity-induced bifurcations (DIBs) can cause a system response to transition from simple periodic to period-doubling or to chaos, and vice-versa [1]. This is often highly undesired. Past research has also found that the symmetry/asymmetry of the freeplay is important and can lead to complex responses in the system [2]. Multi-segmented freeplay occurs when a system has upper or lower freeplay limits in which the stiffness changes a second time. For example, a hinged part that reaches a limit and keeps moving now has a much higher stiffness due to the deformation of the hinge on its mounting area. This represents a freeplay system nearing its mechanical limits. In this work, a strongly nonlinear oscillator system with a multi-segmented freeplay nonlinearity is studied to determine the effects of grazing and grazing/sliding bifurcations on the system's response. System results will be used to observe how the physics of the system may change due to the bifurcations. The system used is the forced Duffing oscillator studied by deLangre [3], with the freeplay modified to be multi-segmented. This freeplay term represents two sets of contact springs in the system, the second set having a higher stiffness (near impact) than the first set.

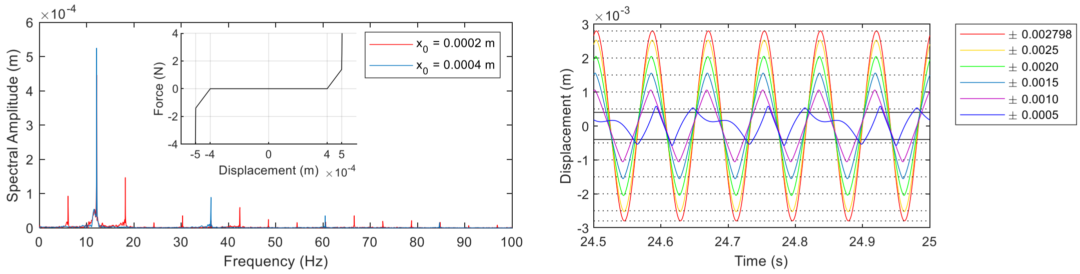

Figure 1: Analysis of response change in the oscillator system. Left: frequency spectra for two initial conditions. Right: steady time histories for various outer-freeplay-boundary values.

Results and discussion

Preliminary results (Figure 1) indicate that, as the outer boundaries are decreased toward the inner boundaries, near-grazing bifurcations occur more often than grazing. Near-grazing/sliding bifurcations are also very common, increasing more when the inner and outer boundaries are close together. Exact grazing does not occur very often, seemingly due to the spring nature of the freeplay. For very close boundaries, different initial conditions (ICs) can cause solutions that are very close in frequency-response but are significantly different in terms of physics. Subharmonic resonances can be activated for some ICs, leading to much more complex behaviors than for other ICs even if all responses are still periodic.

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