An optimal fractional LQR-based control approach applied to a cart-pendulum system

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Abstract. The purpose of this work is to verify the possibility of improving the performance of the control of an inverted cart-pendulum system using an approach based on a fractional-order linear quadratic regulator, thus taking into account the control effort to develop a cost function. For this, the linear Pole Placement method is used to calculate the gains of the controller and the GA optimization to calculate the fractional-orders that enhance performance in comparison to the integer-order integrators. The results show that performance index such as integral square error and settling time of the angular position can be improved by 16% and 20%, respectively.

Introduction

Due to its memory characteristics, fractional-order operators can be used to produce robust controllers for dynamical systems [1]-[2]. Therefore, in this work a state-feedback control system based on a linear-quadratic regulator (LQR) [3] is proposed to stabilize an inverted cart-pendulum system (Figure 1), with the aim of exploring its capabilities to improve the performance of the control system using fractional-states and optimizing the gains and fractional-order exponentes by genetic algorithms (GA).

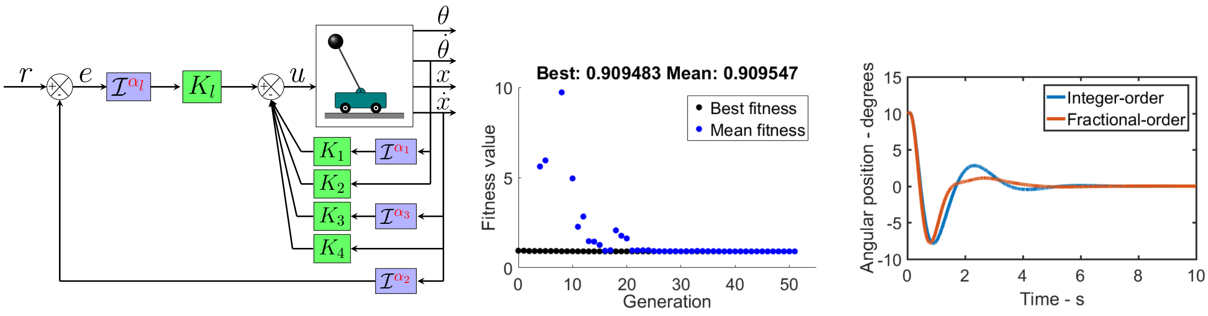

Figure 1: Schematic illustration of the proposed controller for the inverted cart-pendulum system (left). The results of the GA optimization (center) and of the angular position time series with optimal orders for $r(t) = 0$ (right).

The system dynamics evolves according to the following nonlinear model

$$(J + m\,l^2)\,\ddot{\theta} - m\,g\,l\,\sin\theta = -m\,l\,\ddot{x}\,\cos\theta\,, \qquad (m+M)\,\ddot{x} + m\,l\,\ddot{\theta}\,\cos\theta - m\,l\,\dot{\theta}^2\,\sin\theta = u\,, \tag{1}$$

where θ is the angular position and x is the cart displacement. The control system is implemented and executed in Simulink, according to the schematic of the controller diagram shown in Figure 1 (left), where r is the reference input signal; e is the tracking error; u is the control signal; K_1, K_2, K_3, K_4, K_l are the control gains; $\mathcal{I}^{\alpha_1}$, $\mathcal{I}^{\alpha_2}$, $\mathcal{I}^{\alpha_3}$, $\mathcal{I}^{\alpha_l}$ are the integrators, if $\alpha \in \mathbb{Z}$ (integer-order) and if $\alpha \notin \mathbb{Z}$ (fractional order).
The parameters α_1 and α_3 are calculated minimizing with GA, over $0 \leq t \leq \tau$, the LQR-based cost function taking into account the state variables and the control signal

$$\mathcal{J} = \int_0^\tau \Big(x^2(t) + \theta^2(t) + u^2(t) + \dot{x}^2(t) + \dot{\theta}^2(t) \Big)\, dt\,. \tag{2}$$

Results and discussion

The parameters fixed of the controller have been calculated from Pole Placement method, while the fractional exponents α_1 and α_3 have been obtained by GA optimization algorithm. The results are shown in Table 1. The objective function with fractional-order integrators was reduced by 9%, which provided an improvement in the integral square error of x, $\dot{x}$ and θ, the latter with up to 20%. In addition, the settling time of the angular position improved by 16%, as shown in Figure 1 (right), with the comparison of dynamic responses of integer-order and fractional-order. These results show that the fractional-order controller achieved a better response.

Table 1: Performance comparison between integer-order and optimized parameters of the controller.

	integrators order		integral square error				settling time		integral square	$\mathcal{J}$
	α_1	α_3	θ	$\dot{\theta}$	x	$\dot{x}$	θ	x	u	
integer-order	1.00	1.00	0.02	0.19	0.26	0.66	4.87	5.07	23.7	1.00
fractional-order	1.03	0.93	0.017	0.20	0.23	0.51	4.09	5.10	24.4	0.91

References

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