On the physical consistency of evolution laws obtained with sparse regression

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Abstract. This work deals with the use of the sparse identification of nonlinear dynamics (SINDy) to infer the evolution law of the dynamical systems in a data-driven fashion. The Duffing oscillator is used as a benchmark due to the variety and richness of its dynamical behavior. The numerical experiments attempt to identify, whether the method is capable of recognizing the correct evolution law and respecting basic principles of physics such as the balance of momentum and energy. The results seem to show that there is a relationship between the method and basic physical principles.

Introduction

The advent of the information age brought with it the generation of data on a scale never seen before in human history. This amount of data, in combination with machine learning techniques, provides a powerful and appealing way to extract patterns and useful information about different types of phenomena and systems. In the context of dynamical systems, this idea translates into the use of observations (data) to infer the underlying evolution law. In general systems equations are composed of few terms, sparse regression techniques are more proper for this since they allow to obtain interpretable and simpler expressions. This work presents the reconstruction of the evolution law of a Duffing oscillator using a relatively novel technique called sequential threshold least-squares (STLS), which is the central core of the sparse identification of nonlinear dynamics (SINDy) method [1, 2]. The goal is to verify the physical consistency of the evolution law obtained by this machine learning technique, checking if the identified dynamical system respect principles such as the balance of momentum and energy. The Duffing oscillator is used as a benchmark, because of its several applications and well known nonlinear dynamic behavior [3].

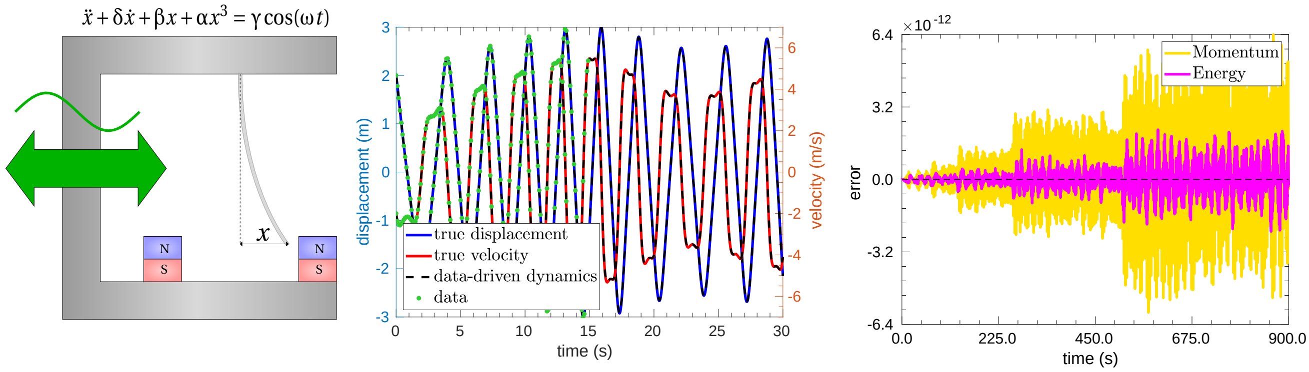

Figure 1: Representation of a Duffing oscillator (left). Comparison between true and identified dynamic response for a forced dissipative oscillator with parameters $\delta = 0.1$, $\alpha = -1.0$, $\beta = -1.0$, $\gamma = 1.0$, $\omega = 2.0$, and initial conditions $x_0 = 2$ and $v_0 = -2$, where the 151 samples equally spaced between 0 and $15s$ are used to train the model (center). Error in the balance of energy and momentum between true and identified dynamics (right).

Results and discussion

Synthetic data from different configurations of the Duffing oscillator are generated. Gaussian noise is introduced in this dataset to reproduce experimental noise and, thus, to check the robustness of the method. Polynomial and trigonometric functions are used as candidate functions to compose the identified evolution law. The SINDy method used to promote the sparsity in the identified dynamics is computationally efficient when compared with other machine learning techniques like deep neural networks [1]. As shown in Figure 1, the method recognized the dynamics with only 151 samples for the dissipative forced Duffing oscillator, obtained in the transient regime. In this Figure it is possible to see a negligible difference in the balance of energy and momentum between the true and identified dynamics. The peaks and valleys of the error in the balance of momentum correspond with the extremes of the position. The local maximums points for the energy balance coincide with the the maximum velocity locations, and the minimum with zero velocity. But looking at these results it seems that there is some correlation between the method and the momentum and energy balance, despise the floating-point noise disturbances.

References

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