Dynamical analysis of a Memristor–Inductor–Capacitor (MLC) nonlinear circuit

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Abstract. We make a dynamical analysis of a memristive circuit consisting of three elements: a passive linear inductor, a passive linear capacitor and an active memristive device (or memristor). The circuit is described by a four parameter system of ordinary differential equations. We show that, for certain parameter values, this system presents polynomial first integrals and invariant algebraic surfaces, and we also study the local behavior of the orbits near equilibrium points. Varying the parameter values, the structures of phase space determined by these invariant sets are broken, giving rise to rich dynamical behaviors. This analysis contributes to better understand the behavior of such a memristive circuit.

Introduction

For more than 150 years, the known fundamental passive circuit elements were the capacitor (discovered in 1745), the resistor (1827), and the inductor (1831). Then, in a seminal paper [1], professor Leon Chua, from University of California at Berkeley, predicted the existence of a fourth fundamental device, which he called a *memristor*, contraction of "memory resistor". Despite this theoretical prediction, the physical realization of a memristor was possible only in 2007, when Stanley Williams and his team at HP Labs constructed a nano-electronic device showing certain characteristics of a memristor [2]. Since then, the interest in the study of memristive circuits increased exponentially, due to its potential applications in several technological areas. Indeed, as memristive circuits exhibit several different nonlinear phenomena, beyond the development of memory technology they play an important role in the development of new nonlinear information processing methods and their implementation in hardware embodiments. Especially, major efforts are being made to develop neuromorphic memristor technology aiming to build complex brain-like computing structures [3]. As the cost of building memristive devices in industrial scale is still very high, the theoretical study of their properties, including computational simulations and emulations, become very important to better understand their behavior.

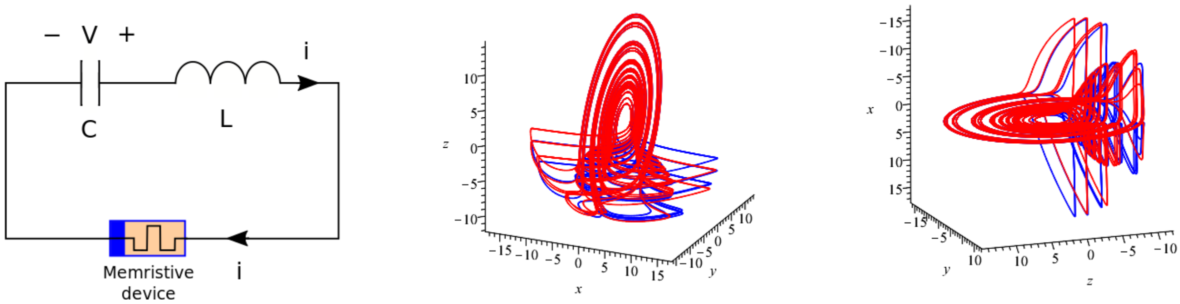

Figure 1: Schematic of a Memristor–Inductor–Capacitor (MLC) circuit and two different views of its nonlinear oscillations.

Results and discussion

In this work we consider a memristive circuit, called *MLC circuit*, consisting of three elements (see Figure 1): a passive linear inductor, a passive linear capacitor and an active memristive device (or memristor). The circuit is described by the following nonlinear system of ordinary differential equations

$$\dot{x} = -x(y+\beta), \qquad \dot{y} = -\omega z - \alpha(x^2-\mu)y, \quad \dot{z} = \omega y, \tag{1}$$

where α, β, μ and ω are control parameters, x, y and z are state variables, which are proportional to the internal state of memristor, to the current and the voltage in the circuit, respectively, and the dot denotes derivative with respect to the time t. The rich dynamics of system (1) was first described and numerically studied by Itoh and Chua in [4]. Here, through an algebraic and numerical analysis we show that, for certain parameter values, system (1) presents polynomial first integrals and invariant algebraic surfaces, which imply the existence of certain global structures in its phase space. We also study the local behavior of the orbits near equilibrium points. Varying the parameter values, these invariant structures are broken and system (1) exhibits rather complex dynamical behavior, as shown in Figure 1.

References

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