Controllability of a two-body limbless crawler on an inclined rough plane

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Abstract. A limbless crawler modeled by two interacting bodies (mass points) on a rough inclined plane is considered. Coulomb's dry friction acts between the bodies and the underlying plane. The control is provided by the force of interaction between the bodies. The issue under study is whether it is possible to bring the system from an arbitrary initial state of rest to an arbitrary terminal state of rest. This issue can be regarded as that of controllability of the system under consideration. The controllability sufficient conditions are proved, and a control algorithm is constructed.

Introduction

The model of the crawler is shown in Figure 1. Here, M and m denote the bodies and their masses; γ is the inclination angle of the plane; $\mathbf{g}$ is gravity field vector, and $\mathbf{f}$ is the control force of interaction acted by body M to body m.

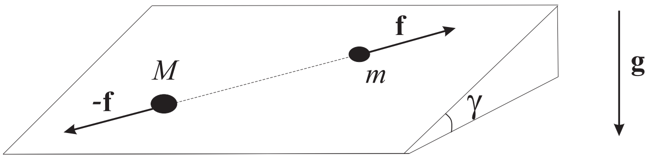

Figure 1: Two-body crawler on an inclined plane.

Denote by k the coefficient of Coulomb's friction between the bodies and the plane. We assume that

$$m < M, \quad \tan\gamma < \frac{M-m}{M+m}k, \tag{1}$$

These relations provide that from any state of rest of the system, body m can be moved, by an appropriate force $\mathbf{f}$, both away from or toward body M, with body M remaining at rest. On a horizontal plane, the two-body crawler can move only along the line that connects the initial positions of the bodies and, hence, the system is uncontrollable. To be controllable on a horizontal plane, a limbless locomotion system must have a more complex structure, see, e.g., [1]. The controllability of the two-body crawler on an inclined plane is explained by a non-zero projection of the gravity force onto this plane. We are interested in the controllability of the crawler in principle, and for this reason, do not impose any constraints of the control force. Moreover, we admit instantaneous changes in the distance between the bodies; in this case, the control force is a derivative of Dirac's delta function with respect to time, multiplied by the product of the respective change in the distance and the system's reduced mass. The system under consideration in uncontrollable, if at the initial state both bodies lie on the common line of maximal slope. In this case, the crawler can move only along this line. The motion of a two-body crawler along a line of maximal slope on an inclined plane is studied in [2].

Results and discussion

It is proved that the system under consideration can be driven from an arbitrary initial state of rest (except for the case where at the initial state the bodies lie on the same line of maximum slope) into an arbitrarily small neighborhood of any prescribed terminal position. An algorithm of the motion is constructed. The motion is implemented by alternating two modes: a slow quasistatic motion of body m, while body M is resting, and a fast motion with instantaneous change in the distance between the bodies. In the quasistatic mode, the motion of body m occurs so slowly that one can assume the equilibrium equations to hold at any time instant. In such a motion, the value of the dry friction force applied to body m is equal to the sliding friction force, and the direction of this force defines the tangent to the trajectory of the quasistatic motion. The fast motion occurs along the line that connects the bodies at the beginning of this motion, the center of mass of the system remaining fixed. These properties of the fast motions are implied by the conservation laws for the linear and angular momenta.

References

[1] Chernousko F.L. (2016) Motion of a body along a plane under the influence of movable internal masses *Doklady Physics* **66**(10): 494-498.

[2] Bolotnik N., Schorr P., Zeidis I., Zimmermann K. (2019) Periodic locomotion of a two-body crawling system along a straight line on a rough inclined plane *ZAMM* **98**(11): 1930-1946.