

## Internal Resonances and Applications in Vibratory Energy Harvesting

Li-Qun Chen

Shanghai Key Laboratory of Mechanics in Energy Engineering, Shanghai Institute of Applied Mathematics and Mechanics, School of Mechanics and Engineering Science, Shanghai University, Shanghai, 200444, China

Email: [chenliqun@hit.edu.cn](mailto:chenliqun@hit.edu.cn)

Internal resonance is a typical nonlinear phenomenon. Internal resonance is usually associated with double jumps, two peaks bending to the left and the right respectively in amplitude-frequency responses. The talk begins with two cases in which 1:2 internal resonance results in the change of hardening and softening characteristics in the amplitude-frequency responses. One case is a pipe conveying fluid flowing in the supercritical speed, and the analysis is based on a discretized model. The other case is coupled cantilevers subjected to magnetic interaction, and the analysis is based on a distributed model. In both cases, with the increase or the decrease of a parameter, multi-scale analysis reveals that double jumps evolve from a jump with softening characteristic and disappear as a jump with hardening characteristic, and the analytical outcomes are supported by numerical simulations. Double jumps with internal resonance may be a possible mechanism to enhance energy harvesting by broadening the harvester working frequency bands. An electromagnetic device with snap-through nonlinearity is proposed as an archetype of an internal resonance energy harvester with double jumps in the amplitude-frequency responses derived from the method of multiple scales. To show the effectiveness, the averaged root-mean-square output voltages are calculated under four kinds of noises, namely, the Gaussian white noise, the colored noise defined by a second-order filter, the narrow-band noise, and exponentially correlated noise. Finally, an L-shaped cantilevered structure laminated with a piezoelectric patch and augmented with frequency tuning magnets is treated analytically, numerically and experimentally. All these works demonstrate that the internal resonance increases the opening bandwidth and the output electricity.

### Bio-sketch of Li-Qun Chen



Li-Qun Chen received his B. Eng (Mechanical Engineering) from Liaoning Science and Technology, Anshan, China, M. S. (Mechanics) from Northeastern University, Shenyang, China, and Ph. D. (Mechanics) from Shanghai Jiao Tong University, Shanghai, China. As a Changjiang Distinguished Professor awarded by the Ministry of Education, PRC in 2008, he is currently working in Shanghai University and Harbin Institute of Technology, Shenzhen. His research interests include nonlinear reduction of vibrations, vibratory energy harvesting, and vibrations of gyroscopic continua. His research was supported by the National Science Fund for Distinguished Young Scholars (2008-2012) and awarded the National Natural Science Prize (2017). He has been one of Most Cited Chinese Researchers (Elsevier) since 2014. He coauthored 4 textbooks in

Chinese, "Theoretical Mechanics" (2006, 2014), "Mechanics of Vibrations" (1999, 2011, 2019), "Nonlinear Vibrations" (2001, 2020), and "Nonlinear Dynamics" (2000), and 2 Springer books, "Chaos in Attitude Dynamics of Spacecraft" (2012) and "Dynamics of Vehicle-Road Coupled System" (2015). Currently, he serves as Associate-Editor-in-Chief of Applied Mathematics and Mechanics (English Edition) and Associate Editor of Nonlinear Dynamics.