Bayesian local surrogate models for the control-based continuation of multiple-timescale systems

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Abstract. Numerical continuation is a popular method for locating and tracking bifurcations in a model. Control-based continuation (CBC) reformulates this for cases when a model is unavailable, allowing experimenters to explore the bifurcation structure of black-box and physical systems [1]. CBC has been demonstrated on a variety of mechanical systems, including a nonlinear energy harvester [2] and a multi-degree-of-freedom system with harmonically coupled modes [3]. Applying CBC becomes impractical when signals cannot easily be represented by the currently-used Fourier discretisation method. Here we propose a surrogate modelling approach, whereby recorded data are replaced by a cheaply evaluated closed-form model. A range of standard discretisation methods can then be applied to this surrogate model.

Introduction

CBC uses feedback control to steer system dynamics towards a desired reference signal. This reference signal is the target of the continuation procedure, and as such, requires discretisation. Fourier discretisation is typically used in CBC, whereby a periodic signal is represented by a truncated Fourier series. However, this method is impractical for the fast-changing signals seen in multiple-timescale dynamics, which contain large amounts of high-frequency energy and require many Fourier coefficients to discretise. Continuation requires a Jacobian, which for CBC must be experimentally computed using finite differences; this is impractically slow when the discretisation is high-dimensional. Furthermore, noise cannot easily be removed from such signals, as a simple low-pass filter will remove both noise and signal. Here we develop a surrogate-modelling approach for CBC, where local models are fitted to experimental recordings at given parameter values. Surrogate modelling is a method whereby the target of an analysis – be it data or a complex nonlinear model – is replaced by a cheaply evaluated surrogate, such that analyses can be performed on the surrogate more readily than on the original problem. Here, surrogates are chosen to reconstruct the original, noise-free signal. A range of discretisation methods can then be applied using the surrogate model, allowing for a more favourable discretisation than would be achieved through direct Fourier decomposition.

Results and discussion

Gaussian process regression (GPR) models are tested on synthetic data, and compared against Bayesian free-knot splines [4]. Stationary and periodic GPR performs well on slowly-changing signals – those which are accurately modelled by a low-order truncated Fourier series – however they fail when fitted to the fast-changing signals that Fourier underperforms on. Bayesian free-knot splines are found to reconstruct both slow- and fast-changing signals to a high degree of accuracy. Figure 1 shows the results of applying free-knot splines, as implemented in [5], to a multiple-timescale signal obtained from a simulation of a Hodgkin-Huxley neuron. The signal is artificially corrupted by normally distributed i.i.d. noise of variance $\sigma^2 = 1.5$. Splines are found to reconstruct the original signal to a high degree of accuracy. The fitted surrogate model offers a closed-form, noise-free description of the system output, which can be discretised for CBC using standard methods.

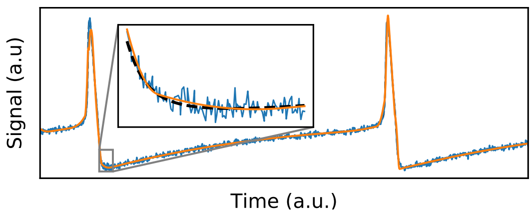

Figure 1: Free-knot splines can reconstruct a signal from noisy observations of a strongly nonlinear system. Black-dotted: true signal; blue: training signal; orange: splines model fit.

M.B. is funded by an EPSRC Doctoral training partnership. L.M. is funded by the Engineering and Physical Sciences Research Council (EPSRC, grants EP/R041695/1 and EP/S01876X/1) and Horizon 2020 (CosyBio, grant agreement 766840). L.R. has received funding from the Royal Academy of Engineering (RF1516/15/11) which is gratefully acknowledged.

References

[1] Sieber J, Krauskopf B. Control based bifurcation analysis for experiments. Nonlinear Dynamics. 2008 Feb 1;51(3):365-77.

[2] Barton DA, Burrow SG. Numerical continuation in a physical experiment: investigation of a nonlinear energy harvester. Journal of Computational and Nonlinear Dynamics. 2011 Jan 1;6(1).

[3] Renson L, Shaw AD, Barton DA, Neild SA. Application of control-based continuation to a nonlinear structure with harmonically coupled modes. Mechanical Systems and Signal Processing. 2019 Apr 1;120:449-64.

[4] DiMatteo I, Genovese CR, Kass RE. Bayesian curve-fitting with free-knot splines. Biometrika. 2001 Dec 1;88(4):1055-71.

[5] Wallstrom G, Liebner J, Kass RE. An implementation of Bayesian adaptive regression splines (BARS) in C with S and R wrappers. Journal of Statistical Software. 2008 Jun 1;26(1):1.