On Generalization of Resonances in Parametrically Excited Systems

Ashu Sharma[*]

[*]*Department of Mechanical Engineering, Auburn University, Auburn, AL, USA*

Abstract. Various types of resonances appear in the dynamics of parametrically excited systems. Resonance conditions for such instability phenomena involve natural frequencies and therefore, they are valid when the amplitude of the parametric excitation term is zero or close to zero. The present work generalizes these resonance conditions so that they are valid in the entire parametric space. This is achieved by obtaining resonance conditions in terms of '*true characteristic exponents*' using different theorems or nonlinear techniques. It is observed that generalized resonance conditions expressed in terms of '*true characteristic exponents*' have similar forms as existing resonance conditions involving natural frequencies.

Introduction

The dynamics of parametrically excited systems are characterized by distinct types of resonances such as parametric resonance, combination resonance, internal resonance, and subharmonic and superharmonic resonances. These instabilities emerge when there is a rational relationship among parametric excitation frequency ω_p, natural frequencies, ω_r ; $r = 1, 2, ...$ and external excitation frequency, ω_f. Since such relationships/ resonance conditions involve natural frequencies, they are valid when the amplitude of the parametric excitation term is zero or close to zero. In order to define resonances for relatively larger amplitudes of the parametric term, it is necessary to construct generalized resonance conditions that are valid everywhere in the parametric space. This is achieved by expressing resonance conditions in terms of '*true characteristic exponents*' which are defined using characteristic exponents and their non-uniqueness property. Unlike characteristic exponents, '*true characteristic exponents*' approach natural frequencies as the amplitude of the parametric term approaches zero. The present study has categorized parametrically excited systems in four classes: linear systems with parametric excitation, linear systems with combined parametric and external excitations, nonlinear systems with parametric excitation and nonlinear systems with combined parametric and external excitations. Each class is investigated separately for different types of resonances and examples are provided to establish the proof of concept.

Results and Discussion

Resonance conditions for linear systems with parametric excitation are constructed using Lyapunov-Poincaré theorem [1]. As an example, a linear mathematical model of a double inverted pendulum is investigated for various types of parametric and combination resonances present in its stability diagram. If true exponents of the system are computed as $\pm\bar{\lambda}$ and $\pm\bar{\beta}$, it is found that parametric resonance occurs when $2\bar{\lambda} = iN\omega_p = N\bar{\omega}_p$ $\left(\text{or } 2\bar{\beta} = N\bar{\omega}_p\right)$; $N \in \mathbb{Z} \setminus \{0\}$ and combination resonance is triggered if $\bar{\lambda} \pm \bar{\beta} = N\bar{\omega}_p$. In the case of linear systems which are subjected to combined parametric and external excitations, Lyapunov-Floquet transformation [1] is utilized to generate a resonance condition. A forced Mathieu equation is examined and it is observed that solution increases unboundedly due to linear resonance when $k_1\bar{\omega}_p + k_2\bar{\omega}_f = \bar{\lambda}$; $k_1, k_2 \in \mathbb{Z}$ is satisfied. The last two categories of parametrically excited systems involve nonlinearity and are investigated using 'time-dependent normal forms (TDNF)' [2, 3]. For nonlinear systems with parametric excitation, TDNF in ref. [2] is employed to obtain resonance conditions. A double inverted pendulum with cubic nonlinearity is examined for internal and combination resonances. It is found that for a 4×4 cubic system internal resonances can be of three types: $\bar{\lambda} = \bar{\beta}$, $\bar{\lambda} = 3\bar{\beta}$ and $3\bar{\lambda} = \bar{\beta}$ and combination resonance occurs when $\bar{\lambda} \pm \bar{\beta} = s\bar{\omega}_p$, $3\bar{\lambda} \pm \bar{\beta} = s\bar{\omega}_p$ and $\bar{\lambda} \pm 3\bar{\beta} = s\bar{\omega}_p$. Here, s is a finite integer. Finally, nonlinear systems with combined parametric and external excitations are studied using the TDNF presented in ref. [3]. A forced Mathieu equation with cubic nonlinearity is investigated for superharmonic and subharmonic resonances. It is found that superharmonic resonances of order 3 occur when $\left(k_1\bar{\omega}_p + k_2\bar{\omega}_f\right) = \pm\bar{\lambda}/3$ whereas subharmonic resonances of order 3 can appear in the nonlinear solution if $\left(k_1\bar{\omega}_p + k_2\bar{\omega}_f\right) = \pm 3\bar{\lambda}$.

References

[1] Yakubovich V.A., Starzhinski V.M. (1975) Linear Differential Equations with Periodic Coefficients-Parts I. John Wiley & Sons, NY.

[2] Sinha S.C., Pandiyan R. (1994) Analysis of Quasilinear Dynamical Systems with Periodic Coefficients via Liapunov-Floquet Transformation. *Int J Non-Linear Mech* **29(5)**: 687-702.

[3] Gabale A.P., Sinha, S.C. (2009) A Direct Analysis of Nonlinear Systems with External Periodic Excitations via Normal Forms. *Nonlinear Dyn.* **55(1)**: 79-93.