Analysis of General Piecewise-Linear Nonlinear Systems Using a Hybrid Analytical-Numeric Computational Method

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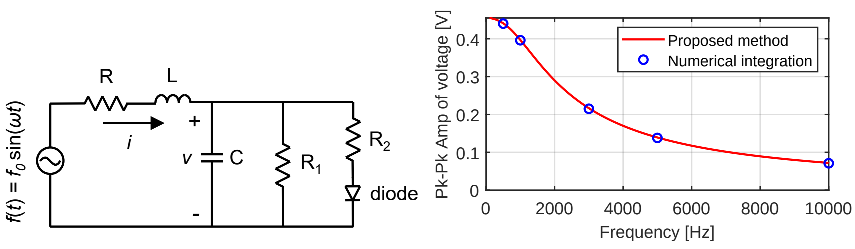
Abstract. In this paper, a new technique is presented for analyzing the steady-state dynamics of piecewise-linear (PWL) nonlinear systems. This new method provides an efficient computational framework for computing the nonlinear vibrational response of general PWL nonlinear systems modeled using state-space representation. The new method combines the analytical and numeric techniques to directly solve for the steady-state response of PWL systems when subjected to periodic forcing. The method is demonstrated on a nonlinear analog circuit with voltage-dependent resistance and a mechanical oscillator with contacting elements.

Introduction

In this paper, an efficient and accurate method for calculating the steady-state response of PWL nonlinear systems is introduced. The new method extends the bilinear amplitude approximation (BAA) method [1, 2], which was originally developed for proportionally damped structural systems, to more general nonlinear systems modeled using state-space representation. The generalized BAA method is based on the idea that the dynamics of a bilinear system can be treated as a combination of linear responses in two time intervals both of which the system behaves as a distinct linear system: (1) the open state and (2) the closed state. The analytical expression of these linear responses can be derived using linear vibration theory. Both geometric and momentum constraints are then applied as compatibility conditions between the states to couple the linear response for each time interval. Since the generalized BAA method is based upon linear methods, the response of the system can be efficiently predicted. In order to extend the BAA method to more general systems, the analytical expression for PWL responses in the state space is derived in this work. The compatibility conditions are also generalized to state space so that the method can be applied to dynamical systems in general engineering areas. The optimization algorithm is then employed to numerically solve for the unknown parameters in the analytical expression and the time that the system spends in each linear state. The method is then demonstrated on a nonlinear analog circuit with voltage-dependent resistance and a mechanical oscillator with contacting elements. The responses computed using the proposed technique is validated with traditional numerical integration. Also, the computational savings obtained by the new method is discussed.

Results and discussion

The proposed method is applied to analyze the dynamic response of a simple nonlinear circuit with a voltage-dependent resistance as shown in Fig. 1(a). This circuit is composed of several components including an inductor (L), a capacitor (C), a diode and several resistors (R, R_1, R_2). Note that the diode allows current to flow in only one direction and hence induces the PWL nonlinearity in the system. Furthermore, the system is driven by an AC voltage with amplitude f_0 and frequency ω. The steady-state response of the system for a particular parameter set is computed using the proposed technique and is shown in Fig. 1(b). The computed responses are also compared with numerical integration solutions. Fig. 1(b) show that the proposed technique is able to accurately capture the dynamic response of the system.



(a) (b)

Figure 1: Comparison of steady-state responses computed using the proposed method and numerical integration. (a) A PWL nonlinear circuit with voltage-dependent resistance. (b) Peak-to-peak amplitude of voltage.

References

[1] Jung, C., D'Souza, K., Epureanu, B.I. (2014) Nonlinear amplitude approximation for bilinear systems. *J. Sound Vib* **333**, 2909–19.

[2] Tien, M.-H., D'Souza, K. (2017) A generalized bilinear amplitude and frequency approximation for piecewise-linear nonlinear systems with gaps or prestress. *Nonlinear Dyn* **88**, 2403–2416.