Characterizing fundamental, superharmonic and subharmonic resonances using phase resonance nonlinear modes

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Abstract. Nonlinear normal modes (NNMs) are often used for the prediction of the backbone of resonance peaks in nonlinear frequency response functions. However, in principle, NNMs require multi-point, multi-harmonic external forcing for their practical realization. The present study proposes a new NNM definition termed phase resonance nonlinear modes. The definition is based on virtual, mono-point, mono-harmonic external forcing proportional to a specific harmonic of the velocity of the forced degree of freedom. Depending on the chosen harmonic, fundamental, superharmonic and subharmonic resonances can be characterized.

Introduction

NNMs can be defined as *a (nonnecessarily synchronous) periodic motion* of the considered unforced and undamped nonlinear mechanical system [1]. This definition was extended to damped nonlinear systems in different studies [2, 3, 4]. However, in all those definitions, the practical realization of a NNM requires multi-point, multi-harmonic forcing, which is difficult to achieve experimentally. In addition, previous works disregarded resonances other than fundamental resonances. Both issues are adressed in this research through a new NNM definition, termed phase resonance nonlinear modes (PRNMs).

Results and discussion

By decomposing the displacement of the forced degree of freedom into its Fourier series, any harmonic can enter in resonance as long as the relation $\frac{k}{\nu}\omega$ is a natural frequency of the mechanical system, which can lead to superharmonic and subharmonic resonances if the ratio k/ν is greater or lower than 1, respectively.
To characterize all those resonances, the PRNM definition injects a virtual forcing signal proportional to the harmonic in resonance of the velocity of the forced degree of freedom $\dot{x}_{\frac{k}{\nu}}$ in the unforced system:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) + k_{nl}x^3(t) - \xi\dot{x}_{\frac{k}{\nu},T}(t - \frac{\nu}{k}\frac{\alpha}{\omega}) = 0 \tag{1}$$

where α is an angle different from 0 if a phase shift other than $\pi/2$ is sought, which can be the case of even harmonic resonances such as, e.g., 2:1 or 1:2 resonances. The forcing term, in turn, ensures quadrature between displacement and forcing, a well-known condition for resonance.

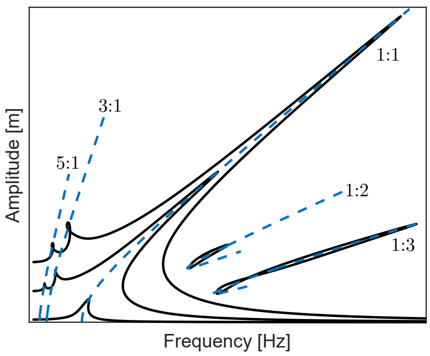

Figure 1: Nonlinear frequency responses and PRNMs of a Duffing oscillator.

In Figure 1, the nonlinear frequency responses of a single-degree-of-freedom Duffing oscillator are represented for different forcing levels. The calculated PRNMs are superposed to this Figure, which demonstrates that they offer an excellent characterization of the backbone of the fundamental resonance but also of the 3:1 and 5:1 superharmonic resonances and of the 1:2 and 1:3 subharmonic resonances.

References

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