Analytical modelling of the dynamics of coupled triads in stratified fluids: Applications to oceanic waves

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Abstract. Triad resonance occurs when three waves in a nonlinear, dispersive medium satisfy special constraints in angular frequencies and wave numbers. Coupled triads, two sets of triad resonance with one common member, are studied in a fluid mechanics context. In contrast with known classical results, coupled triads where all five waves travel in the same direction can arise if density stratification or shear current is present. Analytical formulations are performed for two cases, (a) a continuously stratified fluid with a constant buoyancy frequency, and (b) a two-layer shear flow. The nonlinear dynamics of the first example is investigated by deriving the evolution equations of slowly varying wave packets. In particular, the properties of the system and the dependence on parameters like channel width and the Brunt-Väisälä frequency are explicitly demonstrated. A simpler system with time dependence only is studied to capture the nonlinear physics.

Introduction

Triad resonance (three-wave resonance) is a general physical phenomenon which occurs frequently in the fields of nonlinear optics and water waves. It can be triggered when the wave numbers and wave frequencies of three components satisfy special constraints. Triad resonance for oceanic internal waves has been investigated intensively from different perspectives, such as subharmonic resonance and effects of mean flow and rotation [1,2]. The focus here will be sets of coupled triads, or more precisely, a pair of triads with a common member. The existence of new coupled triads for interfacial and internal waves are highlighted for fluid flows in the presence of stratification or shear current. Examples employed are (a) stratified fluid with a constant buoyancy frequency N_0, and (b) a two-layer fluid with a linear shear. Evolution equations are derived for the first example.

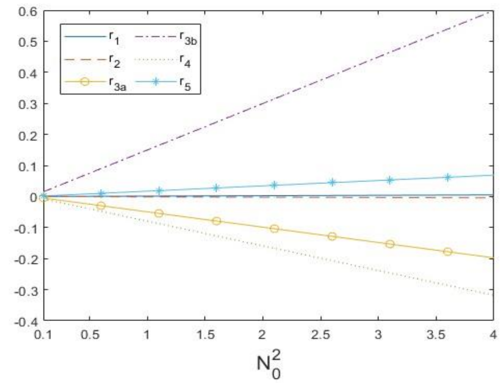

Figure 1: Direct proportional relation between the interaction coefficients 'r' and the buoyancy frequency N for various fluid scenarios. For each value of N, the configuration under investigation is a stratified fluid with constant buoyancy frequency $N = N_0$.

Results and Discussion

The dependence of the interaction coefficients, for various coupled triads with a fixed common member, on the channel height and the buoyancy frequency is elucidated. Theoretical formulations are performed by employing the Madelung representations. Conservation laws for the wave amplitudes similar to the classical Manley-Rowe relations can be established. While the general evolution equations might not be 'integrable', there are special parameter regimes for the possible existence of an additional invariant resembling the Hamiltonian of motion. One intriguing case arises when the difference of the 'dynamical phase' factors of the two triads differs by an integer multiple of 'π'. The approximate governing equation for the amplitude then deviates drastically from the one for an elliptic function of an isolated triad.

References

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