Nonlinear wave propagation in metamaterial waveguides with inertia amplification

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Abstract. In the nonlinear dynamics of periodic systems, the amplitude-dependent dispersion properties of microstructured materials and mechanical metamaterials are attracting increasing interest. The paper investigates the weakly nonlinear dispersion properties of a non-dissipative one-dimensional waveguide with pantographic mass amplification. The effects of the quadratic and cubic inertia nonlinearities on the wavefrequencies and waveforms are analyzed. An asymptotic strategy based on the multiple scale method is adopted. Spectral design considerations are finally pointed out.

Introduction

The dispersion properties of microstructured periodic waveguides are a scientific topic of major interest in nonlinear dynamics. Particularly, the long-dating tradition of research studies concerning the propagation of nonlinear elastic waves in oscillator chains and other periodic structures [1, 2] is being catalyzed by the recent extraordinary development of microstructured materials and mechanical metamaterials [3]. Specifically, a pressing technological demand is boosting the innovation in the emerging field of architected passive media, purposely micro-designed to functionally work as frequency-dependent wave propagators (waveguides) or inhibitors (filters). In order to broaden the existing range of achievable functionalities, the periodic cell of mechanical metamaterials is often enriched with auxiliary flexible microstructures inducing relevant dynamic phenomena, like local resonances, controlled instabilities, gyroscopic couplings, inertia amplifications. The high microstructural flexibility, often accompanied by pantographic mechanisms of amplitude-multiplication and limited dissipation properties, constitutes a natural scenario for the rise of important nonlinear dynamic phenomena.

Pantographic microstructure with inertia amplification

A minimal physical system featured by a periodic pantographic mechanism of inertial amplification can be realized by an infinite one-dimensional non-dissipative chain of interconnected massive atoms (Figure 1). In the periodic tetra-atomic cell, two point masses (*primary atoms*) are elastically coupled to each other and rigidly connected to a pair of auxiliary point masses (*secondary atoms*). The free undamped dynamics of the periodic cell is described by a condensed two-degrees-of-freedom model with quadratic and cubic inertia nonlinearities. The linear and weakly-nonlinear dispersion properties of the periodic system are investigated, according to the Floquet-Bloch theory. In order to achieve analytical – although asymptotically approximate – results, a general perturbation strategy, based on the multiple scale method, is adopted [4, 5]. The effects of the principal mechanical parameters on the amplitude-dependent wavefrequencies and waveforms are investigated. The occurrence of internal superharmonic resonances is considered to discuss the consistency of the asymptotic solutions. Perspectives are outlined about the possibility to govern the nonlinear spectrum of stop and pass bands by employing the mass ratio and pantographic angle as tunable design parameters.

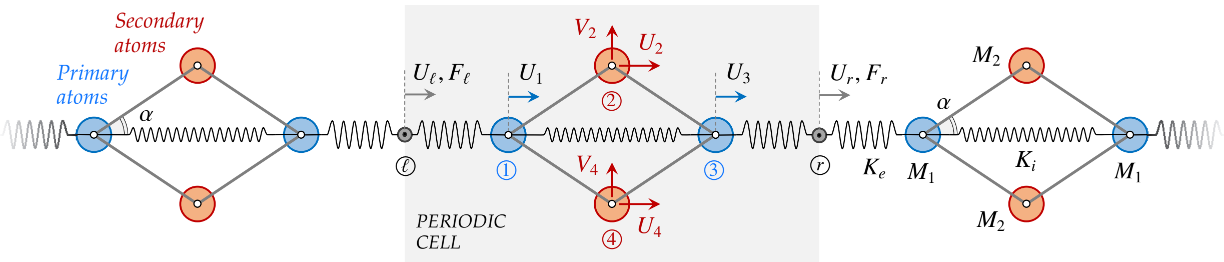

Figure 1: Periodic pantographic waveguide: tetra-atomic microstructure, periodic cell and mechanical properties.

References

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