Forced transversal vibrations of von Karman plates with distributed spring-masses

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Abstract. This research deals with the forced vibrations of rectangular plate incorporating continuously distributed inclusions. The plate model is presented in the form of two mutually penetrating continua, one of them is a classical whereas another one is a set of oscillators forming the auxiliary quasi-continuum. The equations of motion consist of von Karman wave equations for carrying medium and oscillator equation. The simply supported plate is subjected to harmonic forcing. Applying the Galerkin method, the model is reduced to the system of ODE describing the leading eigenmode dynamics of plate vibrations. It is derived analytically the multi-mode periodic solutions, examined their stability and constructed amplitude curves. Numerical simulations revealed the multiperiodic and quasiperiodic regimes as well. The formation of new peculiarities on the amplitude curves, additional regimes and bifurcation scenarios caused by the plate nonlinearity and inclusions' dynamics is shown.

Introduction

Dynamics of structured system typically requires improving the models supported by the classical continuum mechanics in order to incorporate hidden degrees of freedom of structural elements or intrinsic relaxation processes. There are approaches where these system's features are described in the equation of state, but this research considers the structural effects complementing the equations of motion by volumetric forces [1, 2, 3] produced by dynamics of structural elements.

Results and discussion

Based on the works [1, 3], the medium with inclusions [4] is considered as a carrying classical continuum supported by a set of noninteracting oscillators of natural frequency ω. The carrying medium obeys the von Karman plate equation [5] with additional term related to oscillator motion. Then the plate deflection u, the stress function F and the displacement w of partial oscillator satisfy the following equations of motion:

$$\begin{gathered}\rho h u_{tt} + D\Delta\Delta u - h(F_{yy}u_{xx} + F_{xx}u_{yy} - 2F_{xy}u_{xy}) = -m\rho h w_{tt} + \rho h \Omega^2 \gamma \sin \Omega t, \\ \Delta\Delta F = E((u_{xy})^2 - u_{xx}u_{yy}), \quad w_{tt} + \omega^2(w-u) + \tau\omega^2(w-u)_t = 0,\ \Delta = (\cdot)_{xx} + (\cdot)_{yy},\end{gathered} \tag{1}$$

where $D = \frac{Eh^3}{12(1-\nu^2)}$, ρ and h are the plate density and thickness. To specify the problem, the simply supported boundary conditions are subjected. Thus, the purpose of studies are to derive the solutions of this model, find out the bifurcation scenarios, and make conclusion on the cooperative influence of nonlinearity and oscillating dynamics of inclusions. The boundary conditions [5, 6] suggest looking for the solution of (1) in the form: $u = U(t)\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$, $w = W(t)\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$, $F = U(t)^2\Phi(x,y)$, where functions $U(t)$ and $W(t)$ satisfy the following system of ODE:

$$U'' + \frac{D\pi^4(a^2+b^2)^2}{\rho\Omega^2 h a^4 b^4}U + mW'' + \frac{\mu_4}{\Omega^2}U^3 = \frac{16}{\pi^2}\gamma\sin t, \quad W'' + \frac{\omega^2}{\Omega^2}(W-U) + \omega^2\frac{\tau}{\Omega}(W'-U') = 0, \tag{2}$$

$\mu_4 = \frac{E\pi^4}{16\rho a^4 b^4(1-\nu^2)}\left(4\nu a^2 b^2 + (3-\nu^2)(a^4+b^4)\right)$. According to the harmonic balance method, it is derived the amplitude curves based on the solution: $U(t) = \sum_{j=1,3} a_j \sin jt + b_j \cos jt$, $W(t) = \sum_{j=1,3} c_j \sin jt + d_j \cos jt$. Performing the stability analysis of this solution, bifurcation values of forcing frequencies are estimated. When this solution becomes unstable, the system approaches to the regime: $U(t) = z_1 + \sum_{j=1}^4 a_j \sin jt + b_j \cos jt$, $W(t) = z_1 + \sum_{j=1}^4 c_j \sin jt + d_j \cos jt$. Analysis of amplitude curves revealed the different scenarios of oscillation development, hysteretic phenomena and coexistence of attractors. The results obtained can be interesting for the experts in the field of dynamics of heterogeneous (composite) materials, fracture mechanics, metamaterial modeling, development of methods for ultrasonic nondestructive testing.

References

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