Exploring the dynamics of viscously damped nonlinear oscillators via damped backbone curves: A normal form approach

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Abstract. In this work, a novel approximate analytical technique of computing the damped backbone curves resulting by the inclusion of viscous damping is discussed. Traditionally, the analysis of nonlinear systems involves studying the relation between the nonlinear frequency and the resulting vibration amplitudes, this can be performed by computing the conservative (undamped-unforced) backbone curves of the system and comparing them to the numerically computed forced damped frequency responses. The proposed method is based on a variation of Wentzel, Kramers & Brillouin (WKB) and Burton methods and can be directly applied to both SDOF and MDOF systems in order to compute the damped backbone curves of the system.

Introduction

Nonlinear oscillators are widely used for modelling engineering applications including single-degree-of-freedom (SDOF) and multi-degree-of-freedom (MDOF) systems. For instance, single mode approximations of some structural elements such as cables and beams are typical examples of SDOF systems, while MDOF systems involve innumerable applications. One important aspect of such nonlinear systems is the inclusion of viscous damping and how it can affect their dynamical behaviour. Thus, in this work, a novel approximate analytical technique of computing the damped backbone curves resulting by the inclusion of viscous damping is discussed. The analysis of such nonlinear systems involves studying the relation between the nonlinear frequency and the resulting vibration amplitudes, this can be performed by computing the conservative backbone curves of the system and comparing them to the numerically computed forced damped frequency responses, [1]. Although this technique can be highly accurate in the case of undamped and very lightly damped systems, increasing the damping reduces the matching between the conservative backbone curves and the forced damped frequency response curves, hence, less accuracy is achieved in the determination of important features, such as the bifurcation points of the system. In the literature, following the initial work of Krack, [2], several works are introduced to compute the damped backbone curves, usually by adding a fictitious force to the equation of motion in order to introduce an equivalent system by eliminating the viscous damping term.

Results and discussion

The proposed technique is based on Burton work [3] and can be directly applied to any weak nonlinear system in order to generate the damped backbone curves. For instance, the Duffing oscillator with viscous damping is discussed in Figure 1 which shows a comparison between the two backbone curves and the numerically computed forced response (using COCO toolbox in Matlab), the figure clearly shows that, comparing with the numerical forced response, the damped backbone curve is more accurate than the conservative backbone curve.

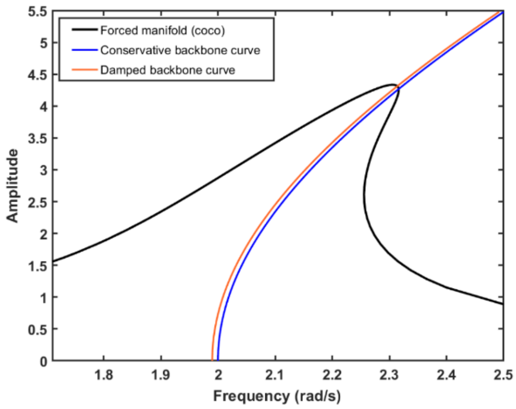

Figure 1: Comparison between conservative and damped backbone curves for Duffing oscillator (ζ=0.1, α= 0.1, ω_n= 2 rad/s)

Using the proposed technique applied symbolically (using Maple software), the work will be generalised to include any order of the polynomial nonlinear terms. Finally, some case studies are discussed in order to investigate the dynamics of nonlinear SDOF and MDOF systems by considering the resulting damped backbone curves.

References

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