Bifurcation and triggers of coupled singularities in the dynamics of generalized rolling pendulums

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Abstract. The results of studying the existence and occurrence of bifurcation of stable equilibrium positions in the dynamics of generalized rolling pendulums, which roll without sliding, along curvilinear paths in a vertical rotating plane around a vertical axis at a constant angular velocity, are presented. It is shown by comparing the two dynamics of this rolling pendulum in cases when the vertical plane, which contains the curvilinear rolling route, is stationary and when it rotates, the angular velocity of rotation of that plane has the property of a bifurcation parameter. When this angular velocity is equal to zero in the rolling dynamics of the rolling pendulum, there are no triggers of coupled singularities, and when it is different from zero in each stable position of the dynamics, that is, a singular point of the stable centre type, bifurcation occurs.

Introduction

The In a series of author 's published articles, between them and papers [1-3], some partially obtained particular results of studying the dynamics of rolling pendulums along curvilinear paths in stationary or rotating vertical planes around a vertical axis at a constant angular velocity, with corresponding phase portraits, are presented. A series of phase portraits with different structures of singular points and phase trajectories, especially those of separatrix, are presented. In the last manuscript [1], special attention is paid to the occurrence of bifurcation [4, 5] of stable equilibrium positions and the existence of triggers of coupled singularities.

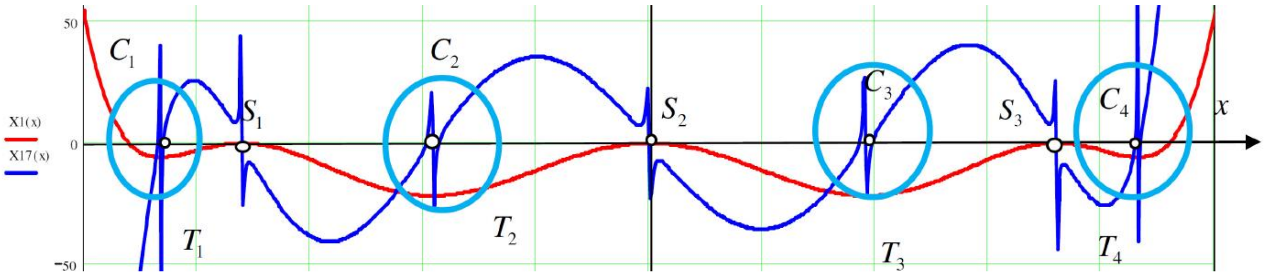

Figure 1: Graphs of the curvilinear route, as well as the frequency functions of the nonlinear rolling dynamics of a rigid heavy thin disk, in a rotating vertical plane with a constant angular velocity around the vertical axis defined by the equation $y = f(x) = kx^2(x^2-a^2)^2(x^2-b^2)$.

Results and Discussion

In this paper, attention is paid to a more detailed analysis of the characteristic equation of dynamics of the generalized rolling pendulum, which was performed in [1] in the form (see Figure 1):

$$f'(x)\left\{1-r\frac{f''(x)}{\left[1+[f'(x)]^2\right]^{\frac{3}{2}}}\right\}-\frac{2\kappa}{3g}\Omega^2\left\langle x-\frac{rf'(x)}{\sqrt{1+[f'(x)]^2}}\right\rangle\left\langle 1-\frac{rf''(x)}{\left[1+[f'(x)]^2\right]\sqrt{1+[f'(x)]^2}}\right\rangle=0$$

and in which: $y = f(x) = kx^2(x^2-a^2)^2(x^2-b^2)$ is equation of the curvilinear path, where a, b and k are known constants, and with the following relation $a<b$, κ the rolling coefficient, r the radius of the circle of the body of the pendulum by which the pendulum rolls along curvilinear paths, Ω the angular velocity of rotation of the vertical plane about the vertical axis, and in which the curvilinear rolling route of the rolling pendulum. It is pointed out that there is a mathematical and qualitative analogy of the properties of the dynamics of a rolling pendulum with the movement of a heavy point along a same form of curvilinear path in a rotating vertical plane around a vertical axis at a constant angular velocity. Two theorems on the existence of triggers of coupled singularities and bifurcations in phase trajectory portraits of nonlinear dynamics of the generalized rolling pendulums along curvilinear trace are defined and proved.

References

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