

# Full-order frequency-domain simulations of nonlinear piezoelectric MEMS

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**Abstract.** We report large scale harmonic balance finite element simulations of piezoelectric MEMS micromirrors. Nonlinear piezoelectric effects induced by the large actuation voltages are modeled by introducing a novel technique to account piezoelectric nonlinearities in thin films. Numerical results are validated with experimental data.

## Introduction

In recent years piezoelectric materials have become the preferred choice for actuation in microelectromechanical systems (MEMS). These materials provide higher forces and enhanced linearity compared to electrostatic or magnetic actuation, making them appealing for the design of systems that undergo large drive displacements, e.g. scanning micromirrors [1]. In this work, the Harmonic Balance Finite Element Method (HBFEM) is exploited to compute stable periodic solutions of MEMS micromirrors actuated with thin films of piezoelectric materials. Geometrical nonlinearities are addressed using the standard finite transformations framework, while piezoelectric nonlinearities are accounted by introducing a novel technique to model piezoelectric effects in films excited at large voltage amplitudes. The frequency response function obtained from the numerical simulations are compared with experimental data.

## Results and discussion

We introduce the following equation to model the nonlinear dynamic response of piezo MEMS subjected to finite transformations and nonlinear piezoelectric effects:

$$\int_{\Omega_0} \rho_0 \ddot{\mathbf{u}} \cdot \mathbf{w} d\Omega_0 + \int_{\Omega_0} \mathbf{T} : \nabla^T \mathbf{w} d\Omega_0 = P^2(t) \int_{\Omega_P} c_{31}(\varepsilon_{11}[\mathbf{w}] + \varepsilon_{22}[\mathbf{w}]) + c_{33}\varepsilon_{33}[\mathbf{w}] d\Omega_P \quad \forall \mathbf{w} \in \mathcal{C}(\mathbf{0})$$

where  $\Omega_0$  is the domain in reference configuration,  $\rho_0$  initial density,  $\mathbf{u}$  displacement,  $\mathbf{w}$  suitable test function,  $\mathbf{T}$  first Piola-Kirchhoff stress tensor,  $P$  polarization vector component orthogonal to the film,  $t$  time,  $\varepsilon_{ij}$  components of the symmetric part of the gradient of  $\mathbf{w}$ , and  $c_{ij}$  coefficients derived from the material properties.  $\Omega_P$  denotes the piezoelectric films volume. The left hand side of the equation corresponds to the classical kinetic and internal energy for mechanical systems, while the right hand side identifies the piezoelectric forces exerted by the actuation films under the assumption of uniform polarization, as derived from the Landau-Devonshire theory of ferroelectrics. The model relies on the a priori knowledge of the polarization history  $P(t)$  of the film that can be measured with experiments or computed through simulations of the polarization evolution within the film [2].

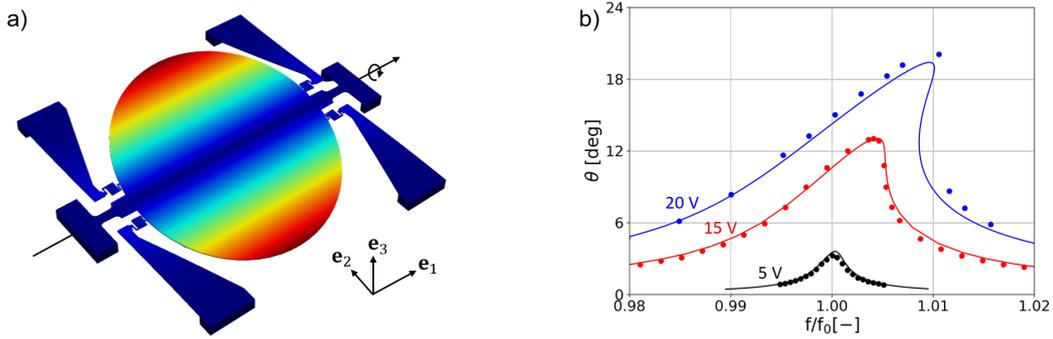


Figure 1: Eigenmode shape of the structure (a) and comparison between the proposed model (continuous line) and experimental data (dots) for different voltage amplitudes (b).

A fifth order HBFEM expansion is here applied to compute periodic solutions of a MEMS micromirror. The device is actuated in its torsional mode, reported in fig.1(a). Numerical results are compared with experimental data in fig.1(b) for three actuation voltage amplitudes: 5, 15, and 20 V. The comparison shows an excellent agreement between the proposed model and the experimental data.

## References

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