Global analysis and experimental dynamics of the 2:1 internal resonance in the higher-order modes of a MEMS microbeam

Laura Ruzziconi*, Nizar Jaber**, Lakshmoji Kosuru***, Mohammed L. Bellaredj*** and Mohammad I. Younis ***

**Faculty of Engineering, eCampus University, 22060 Novedrate, Italy*

***King Fahd University of Petroleum and Minerals (KFUPM), 31261 Dhahran, Saudi Arabia*

****King Abdullah University of Science and Technology (KAUST), 23955-6900 Thuwal, Saudi Arabia*

Abstract. The present study investigates the dynamics of a MEMS device in the neighborhood of the third natural frequency, where the system exhibits the activation of the 2:1 internal resonance with the fifth mode. The main features are analyzed extensively, both experimentally and theoretically. Constantly comparing with the experimental data, the response is examined from a global perspective. The attractor-basins phase portraits report on the metamorphoses occurring as proceeding from the principal resonance to the 2:1 internal resonance, up to the disappearance of the attractors.

Introduction

Many different nonlinear features may arise in MEMS/NEMS [1], whose potential is explored in-depth for realizing novel devices and achieving superior performances [2]. Increasing interest is devoted to the nonlinear interactions among different vibration modes [3]. Within this framework, we focused on the dynamics induced by the 2:1 internal resonance in a microbeam-based MEMS device electrically actuated.

(*a*)

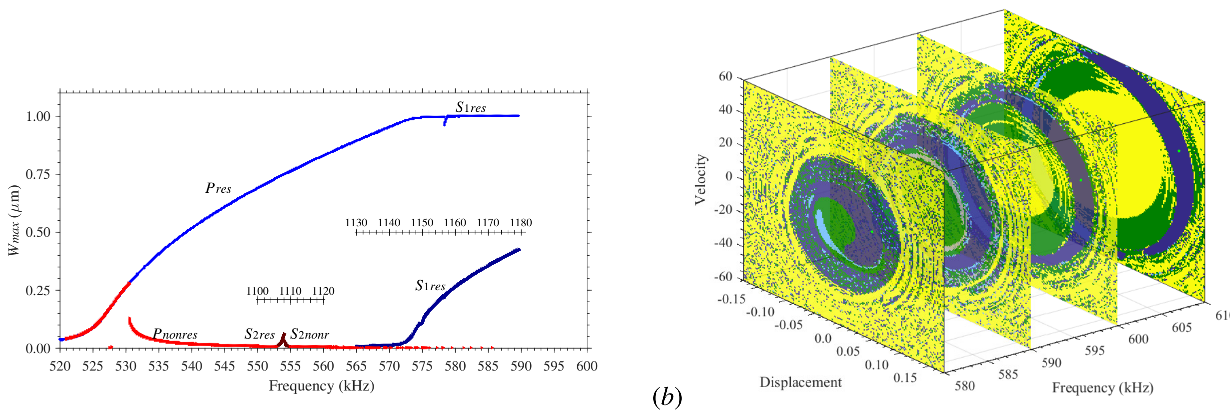

Figure 1: (*a*) Experimental FFT diagram at V_{AC} = 21.5 V. Forward and backward sweep are in blue and dark blue for the third mode, in red and purple for the fifth, respectively. (*b*) Progression of attractor-basins phase portraits around S_{1res}, in the fifth-mode plane.

The 2:1 internal resonance dynamics

An extensive experimental investigation is conducted. Focusing on the resonant branch, the FFT in Fig. 1*a* extracted from the experimental time history shows the third mode dynamics progressively increasing (P_{res}) and successively settling and exhibiting a practically constant plateau (S_{1res}), which denotes saturation of the third mode contribution. Concurrently, the fifth mode activates, increasing its amplitude rapidly, while bending toward the right. Similarly occurs along the non-resonant branch, although smaller amplitudes are achieved.

Systematic simulations are developed based on 2 degree of freedom Galerkin reduced-order model accounting for both the third and fifth mode dynamics, where the electric force term is integrated numerically. Simulations alert that the dynamics observed in the experimental frequency sweeps are part of a more complex scenario, where different attractors appear and coexist. The progression of the attractor-basins phase portraits is examined. The analysis is conducted both in the third-mode plane and in the fifth-mode one. In Fig. 1*b*, at the 2:1 internal resonance the section drawn in the fifth-mode plane around S_{1res} shows its basin (blue) and the concurrent appearance of the basin of $S_{1nonres}$ (green). They spiral around each other and their cores cover a large part of the area previously belonging to P_{res}. The branch $S_{1res-ph}$ (cyan) appears and its development is similar to the basin of S_{1res}. Increasing the frequency, the basin of P_{nonres} (yellow), progressively grows at the centre of the phase portrait, increasing the eccentricity of the other basins and gradually moving them toward the escape. The shape and wideness of each basin are examined, alerting on the criticality related to the 2:1 internal resonance activation. The robustness of each attractor is discussed as reference for the design stage.

References

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