Hidden attractors in 4D discontinuous piecewise linear Memristor oscillators

Javier Ros*, Enrique Ponce* and Andrés Amador**

*Dep. de Matemática Aplicada, E. T. S. Ingeniería, Camino de los Descubrimientos, 41092 Sevilla, Spain

**Dep. de Ciencias Naturales y Matemáticas, Facultad de Ingeniería y Ciencias, Pontificia Universidad Javeriana-Cali, Santiago de Cali, Colombia

Abstract. The so-called multiple focus center bifurcation (MFCC), leading to the appearance of a topological sphere foliated by stable periodic orbits, was recently studied for some discontinuous 4D Memristor oscillators [2]. Extending the quoted work, we study here possible degenerate MFCC bifurcation points. Previous analytical results, showing the existence of a saddle-node bifurcation curve for periodic orbits that emanates from such points of degeneration, allow to prove in a rigorous mathematical way the existence of a parameter region where stable limit cycles coexist with equilibrium points, being all of them also stable. Thus, such stable limit cycles are instances of hidden attractors.

Introduction and main results

An attractor is called a hidden attractor if its basin of attraction does not intersect with some neighborhood of any equilibrium point; otherwise, it is called a self-excited attractor [4]. Self-excited attractors can be easily detected and visualized, since the basin of attraction contains points near an unstable equilibrium; it suffices to follow any orbit starting in a neighborhood of such equilibrium. Since the basin of attraction for a hidden attractor is not connected with any equilibria neighborhood, the search and visualization of hidden attractors in the phase space becomes a challenging task [1].

In this work we consider the canonical discontinuous fourth-order memristor oscillator given in [2, 3]

$$\dot{y}_1 = y_4, \quad \dot{y}_2 = y_3 - y_4, \quad \dot{y}_3 = -\beta y_2 + \gamma y_3, \quad \dot{y}_4 = y_2 - W(y_1) y_4, \tag{1}$$

where $\beta, \gamma > 0$, $W(y_1) = b$ if $|y_1| > 1$ and $W(y_1) = a$ if $|y_1| < 1$. System (1) admits the first integral $H(y_1, y_2, y_3, y_4) = \beta y_4 + \beta q(y_1) - \gamma y_1 - \gamma y_2 + y_3$, where the continuous piecewise linear function $q(x) = \int_0^x W(s)\, ds$ is involved. The equilibrium points of system (1) are all the points in the y_1-axis. Our main result for the 4D discontinuous system (1) guarantees, under certain hypotheses, the existence of a hidden attractor.

Theorem 1 *For system* (1) *with* $a < 0 < b$, $1 < \beta < \phi$, *where* $\phi = (1 + \sqrt{5})/2$ *is the golden ratio, consider in the two-parameter plane* (a, γ) *the curve of MFCC points* $\gamma_+(a) = (a^2 + \beta)/2a + \sqrt{((a^2 + \beta)/2a)^2 - 1}$ *and its point* $-p = \left(-\sqrt{\beta}, -\sqrt{\beta} + \sqrt{\beta - 1}\right)$*, associated to* $a = -\sqrt{\beta}$*. From the point* $-p$ *a curve* δ_{sn} *of saddle-node bifurcations of periodic orbits emanates, see Fig 1(b), so that for parameter values between the curves* δ_{sn} *and* γ_+*, at least for the invariant 3D manifold associated to* $H(y) = 0$*, there are two periodic orbits, one stable and another unstable, and only a stable equilibrium point; that is, there exists a hidden attractive limit cycle, see Fig 1(a).*

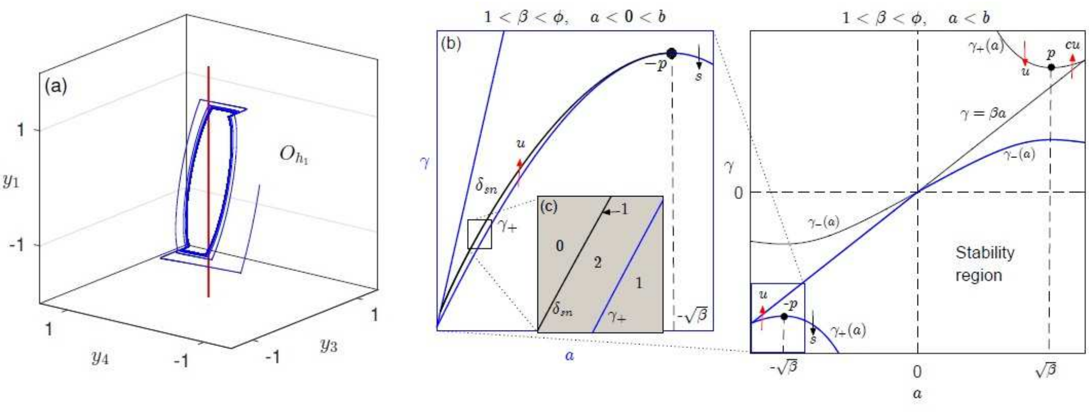

Figure 1: Parameters are $\beta = 1.2$, $a = -1.265$, $\gamma = -1.58$, and $b = 20$. (a) In blue, one of the hidden attractive limit cycles predicted by Theorem 1, which uses the three linear zones of system (1); in red, the stable equilibrium points. (b) Magnification of the involved zone of the two-parameter plane (a, γ). Digits between curves in the grey box stand for the number of periodic orbits there.

References

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