Frequency locking, Quasi-periodicity and Chaos due to special relativistic effects

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Abstract. We study quasiperiodic and frequency-locked states that can occur in a sinusoidally driven harmonic oscillator in the special relativistic regime. We show that the natural frequency shifts with the increase in relativistic effects, thus forming both rational and irrational ratios with the driving frequency. We report the occurrence of frequency-locked states for rational ratios and quasiperiodic states for irrational ratios.Also, the relativistic corrections cause harmonics and multi-stable states due to the consequent nonlinearity.

Introduction

Majority of the studies in nonlinear mechanical systems deals with nonrelativistic cases and chaos exists in such systems due to nonlinearity in the potential function. The study of chaos in relativistic systems is an interesting area of research both for its own nature as well as for its applications in many experimental contexts where particle oscillations occur at effectively high velocities [1]. While it is reported earlier that chaos can appear in most integrable classical systems due to special relativistic corrections to the dynamics[2, 3, 4, 5] , the details of the route to chaos, occurrence of quasiperiodic and frequency-locked states are still not fully understood.

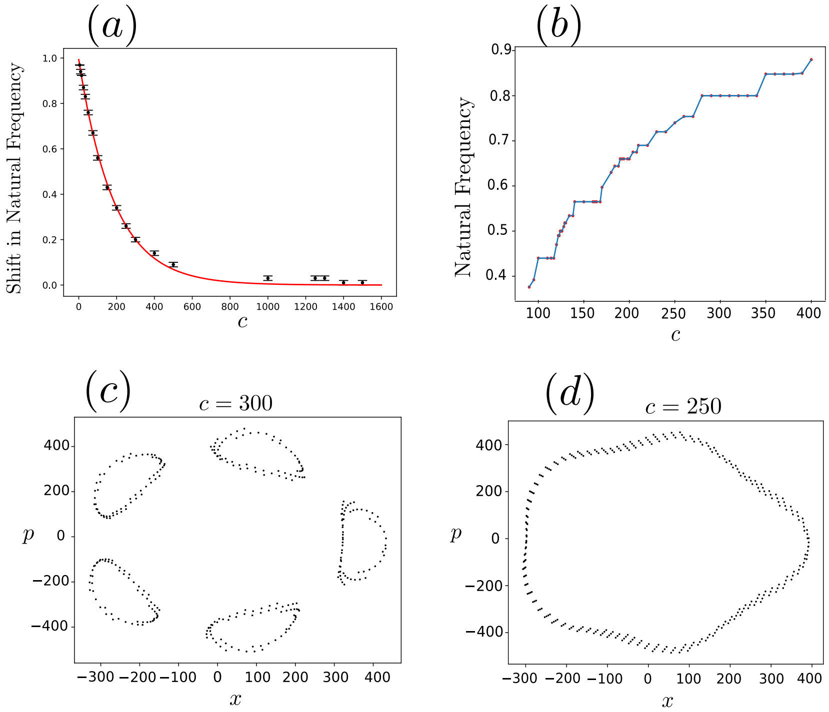

Figure 1: (a) Shift in the natural frequency of the oscillator due to relativistic effects, (b) Devils' stair case indicating frequency locked steps (c) frequency-locked state for c=300 and (d) quasiperiodic state for c=250

Results

In this work, we present a detailed study on the transitions in the dynamics in a one-dimensional forced harmonic oscillator as the effects of the relativistic corrections are incorporated. We show that the natural frequency of the system decreases due to increasing relativistic effects (Fig. 1(a)) and thus can get into rational and irrational ratios with the driving frequency as the velocity of light, c is tuned.Using numerical simulations,we explicitly show the occurrence of frequency-locked states for rational ratios(Fig. 1 (b),(c)) and quasiperiodic states for irrational ratios(Fig. 1(d)).

References

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