Forced Predator-Prey System Employed for Bone Mechano-transduction Description

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Abstract. Based on bone mechanobiology research, this paper develops computational analytical models in order to address and better understand mechano-transduction - the molecular mechanisms by which bone cells sense and respond to mechanical signals. Downstream signalling in response to periodic excitation was modelled by cell population system in order to better represent and predict long-term behaviour and consequences of bone cell loading. The generalized S-system is in charge and is solved deterministically together with its stochastic analogue (Gillespie algorithm) used for noise check and system behaviour dynamics analysis. In-silico experimenting with a number of responding cells which is up to or around a certain threshold allows us to distinguish and describe different dynamics and relations between involved cells. This research clearly shows the indispensability and beneficial effects of external excitation on balanced and regular bone cell activities.

Introduction

The complex and important process of bone cellular communication and respond to external stimuli which contribute to bone structural and geometrical optimal adaptation to bearing leads, has been widely modelled by biological in-vivo/vitro experiments last decades, among many others see, for example Ref. [1]. However, these experiments strive to be more efficient and effective so that in-silico experiments with developed mathematical models of processes take important and valuable places. This study represents a deterministic and stochastic analysis of the forces generalized Predator-Prey (S-System) system that represents of bone cellular transduction of external periodic signals. We explore this system through its homogeneous coupled ordinary nonlinear differential equations,

$$\frac{du_i}{dt} = \alpha_i \prod_{j=1}^{2} u_j^{g_{ij}\cos\theta t} - \beta_i u_i + F_i cos\omega t, \;\; i = 1, 2, \dots, n \qquad (1)$$

From system (1) we are able to extract the stoichiometric creation and degradation relations [2], and present its probabilistic analogue:

$$u_i + u_j \xrightarrow{\alpha_i} 2u_i + u_j, u_i \xrightarrow{\beta_i} \emptyset, i = 1, 2, \dots, n \text{ and } j \neq i. \qquad (2)$$

Continuum model (1) assume that the simulated populations are large enough that a continuum approximation is valid. In such cases the stochastic and deterministic descriptions are equivalent as noise reduces relative to population size. However, in the bone creation-degradation application, which these equations describe, cell population numbers often fall below 10 cells. Thus, the stochastic description is more apt to include the present noise effects [3]. Furthermore, a small change in the parameters of an external process can shift the system over the bifurcation [4], so that the stable steady state becomes unstable, or ceases to exist. It is, therefore, reasonable to perform proposed stochastic simulations of system (2) that reveal the qualitatively different dynamic behaviour.

Results and discussion

The analogous probabilistic model of fifth order (2) cannot be analyzed so easily, thus, we turn to simulation. Namely, stochastically simulating the system 1000 times allows us to extract probabilistic distributions of the outcomes, which capture the noisy features of cell division and death, especially in the transition regimes on the beginning and ending of the processes. Also, in-silico experimenting with a number of responding cells which is up to or around a certain threshold allows us to distinguish and describe different dynamics and relations between involved cells. The external signal can be considered as an additional term affecting the number of responding cells or as the functional periodicity of power low coefficients affecting autocrine signalling of forming cells. This research clearly shows the indispensability and beneficial effects of external excitation on balanced and regular bone cell activities, since the total content of newly formed bone is larger under the periodic loading influence.

References

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