Conserving kinetic energy while identifying nonlinear reduced-order models

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Abstract. Several methodologies have been developed to *indirectly* compute reduced-order models (ROMs) of geometrically nonlinear structures modelled using commercial finite element (FE) software. Currently, such methods are unable to capture the in-plane kinetic energy of the FE model, leading to poor results when applied to structures which can undergo large in-plane displacements. In this work, we show how kinetic energy can be accounted for in the reduced dynamics, and demonstrate the accuracy of the proposed method using a cantilever beam.

Introduction

Nonlinear reduced-order modelling is used to alleviate the, often prohibitively large, computational cost associated with the detailed dynamic analysis of highly complex FE models of engineering structures. Indirect reduction methods identify ROMs using a static solution dataset of the full-order model, and can be used in conjunction with commercial FE software, where the underlying full-order EOMs are inaccessible [1]. In this regard, the *Implicit Condensation and Expansion* (ICE) method can efficiently capture the response of the full-order model using few DOFs [2, 3], but its applicability is typically limited to beam or plate-like structures with fixed or pinned boundary conditions, which allow for limited in-plane displacements. In this work, we aim to extend the ICE method such that it can be applied to a wider range of structures.

Results and discussion

We employ a Lagrangian approach to show how the potential as well as the kinetic energy of the full-order model can be accounted for in the reduced dynamics. This introduces new terms to the reduced EOMs, including higher-order stiffness terms [4], as well as nonlinear inertia and gyroscopic terms, which emerge from the kinetic energy of the in-plane modes. We show that the proposed method aligns with the concept of nonlinear normal modes defined as invariant manifolds in phase space [5, 6], and demonstrate how the Lagrangian-based reduction is equivalent to a projection onto a nonlinear manifold [7]. We compute ROMs of a cantilever beam, modelled in Abaqus, and show how our proposed method can significantly increase the accuracy of the response frequency predictions, relative to the ICE method (figure 1).

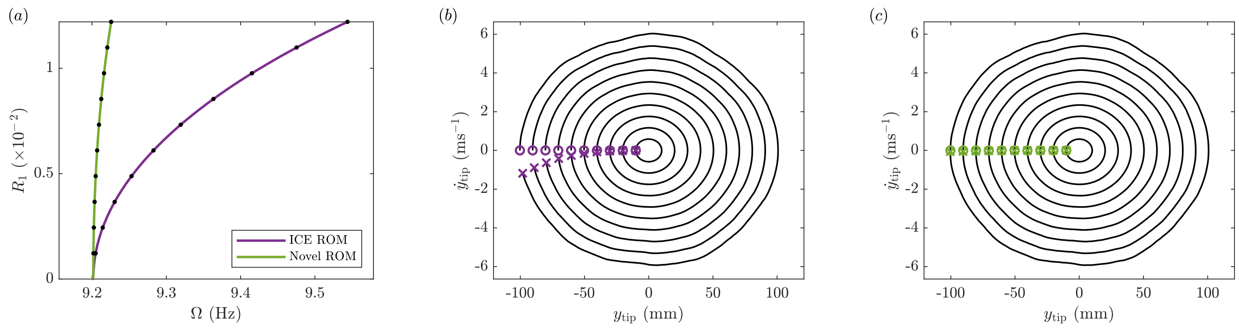

Figure 1: (a) Backbone curves of the ROMs obtained using the ICE method and our proposed method. (b-c) Different free response runs of the FE model, where the initial conditions and the corresponding period of integration are predicted by (b) the ICE ROM and (c) the proposed ROM. The initial FE states and the FE states after one ROM period, are marked with circles and crosses, respectively—when these coincide, the response is periodic. Conversely, an incomplete loop in phase space indicates an overestimate of the response frequency.

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