Influence of circumferential discontinuity of an elastic foundation on the nonlinear dynamics of cylindrical shells with functionally graded material

Jonathas K. A. Pereira*, Renata M. Soares*, **Frederico M. A. Silva*** and Paulo B. Gonçalves**

**School of Civil and Environmental Engineering, Federal University of Goiás, Goiânia, Brazil*

***Department of Civil Engineering, Pontifical Catholic University, Rio de Janeiro, Brazil*

Abstract. In this paper we study the nonlinear behavior of functionally graded (FG) cylindrical shells on an elastic foundation with circumferential discontinuity. For this, a reduced-order model was based on a perturbation method to describe the transversal displacement field, considering the influence of circumferential discontinuity of the elastic foundation. The vibration modes that most influence the linear response of the problem are identified, and these modes are used as an initial solution of the perturbation method to obtain the modal solution. Nonlinear equations of motion are obtained using Donnell's nonlinear theory and they are discretized using the Galerkin method. A parametric analysis is performed and backbones and resonance curves of the FG cylindrical shell are obtained, showing the influence of discontinuity on the nonlinear oscillations.

Introduction

Few studies on the nonlinear dynamics of cylindrical shells with a discontinuous elastic base are found in the literature. The linear behavior due to the discontinuous elastic foundation was studied by Amabili and Dalpiaz [1] considering isotropic shells. Sheng el al [2] studied the nonlinear vibrations of cylindrical shells with FGM on elastic foundation and Silva et al [3] studied the nonlinear vibrations of FGM shells on an elastic foundation with longitudinal discontinuity. Based on these previous work [3], a consistent transversal displacement field that contains the main modal coupling is derived to describe the nonlinear behavior of FG cylindrical shell resting in discontinuous elastic base.

Results and discussion

Consider a transversally excited cylindrical shell of a radius R, length L and thickness h, made of a FG material varying along the thickness. The shell is considered to be on an elastic foundation according of Winkler type with circumferential discontinuity represented by a Heaviside function. A cylindrical shell of radius $R=0.6m$, length $L=0.6m$ and thickness $h=0.003m$ is adopted. The basic materials of FG shell are nickel ($E = 2.051\times10^{11}$ N/m², ρ = 8900 kg/m³ and ν = 0.3) and ceramic ($E = 3.222\times10^{11}$ N/m² , ρ = 2370 kg/m³ and ν = 0.24) with a sandwich type distribution [3]. The cylindrical shell is considered simply supported and the angle of the elastic foundation is $\theta = 45^{\circ}$ and is distributed throughout the length L of the cylindrical shell. The stiffness of the elastic base is $K_W = 7.255\times10^{6}$ N/m². From the frequency spectrum of circumferential modes, it is verified that the main linear vibration modes generate a reduced model with 2 degrees of freedom in the circumferential direction. Using these main modes as the seed solution in the perturbation method, a consistent reduced order model to describe the transversal displacement field was obtained. Figure 1a shows the convergence for different reduced order models of the backbones while Fig. 1b presents the resonance curves for the shell with its Poincaré section diagram showed in Fig. 1c. These results demonstrate that the discontinuity of the elastic foundation can be adequately represented by the reduced models developed by the authors.

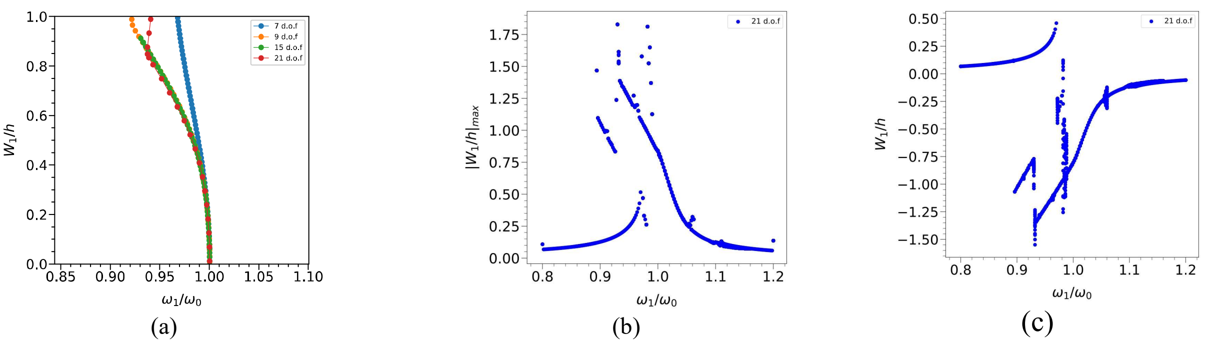

Figure 1: (a) backbone curves for a FG cylindrical shell considering different reduced order models. (b) resonance curves and (c) Poincaré section diagram for a FG cylindrical shell transversally excited for 21 d.o.f model, considering load amplitude equal 5.000 N/m² in the shape of fundamental vibration mode.

References

[1] Amabili, M.; Dalpiaz, G. (1997) Free Vibrations of Cylindrical Shells with Non-Axisymmetric Mass Distribution on Elastic Bed. Meccanica, **32**, 71–84.

[2] Sheng, G. G.; Wang, X.; Fu, G.; Hu, H. (2014) The Nonlinear Vibrations of Functionally Graded Cylindrical Shells Surrounded by an Elastic Foundation. Nonlinear Dyn, **78** , 1421–1434

[3] Silva, F. M.A.; Rodrigues, P. C.; Gonçalves, P. B. Nonlinear Oscillation of a FG Cylindrical Shell on a Discontinuous Elastic Foundation. In: New Trends in Nonlinear Dynamics. Springer, Cham, 2020. p. 79-88.