Analysis of Periodic and Quasi-Periodic Orbits of a Hysteretic Oscillator

Tamás Kalmár-Nagy, János Lelkes

Department of Fluid Mechanics, Faculty of Mechanical Engineering, Budapest University of Technology and Economics, Budapest, Hungary

Abstract. A hysteretic relay oscillator with harmonic forcing is investigated. Periodic excitation of the system results in periodic, quasi-periodic, and unbounded behavior. The initial conditions giving rise to period-one solutions were obtained analytically. The regions of initial conditions leading to quasi-periodic and unbounded solutions were determined by numerical simulations.

Introduction

Hysteresis type nonlinearities are common for various types of systems, including mechanical, electrical and biological systems [1]. Krasnosel'skii and Rachinskii [2] investigated the bifurcations of forced periodic oscillations for systems with Preisach hysteresis. Kalmár-Nagy et al. [3] studied a single degree-of-freedom forced hysteretic system without linear restoring element. The equation of motion of the forced hysteretic system with a linear restoring element can be written as

$$\ddot{x}(t) + x(t) + F\left[x(t)\right] = A\cos(\omega t + \phi_0), \quad A \geq 0, \omega > 0, \quad \phi_0 \in (-\pi, \pi], \tag{1}$$

where A, ω, and ϕ_0 are the amplitude, frequency, and phase of the forcing, respectively. In this model we use a symmetric hysteresis operator

$$F\left[x(t)\right] = \begin{cases} -1, & x(t) \leq -1, \\ e, & -1 < x(t) < 1, \\ 1, & x(t) \geq 1. \end{cases} \tag{2}$$

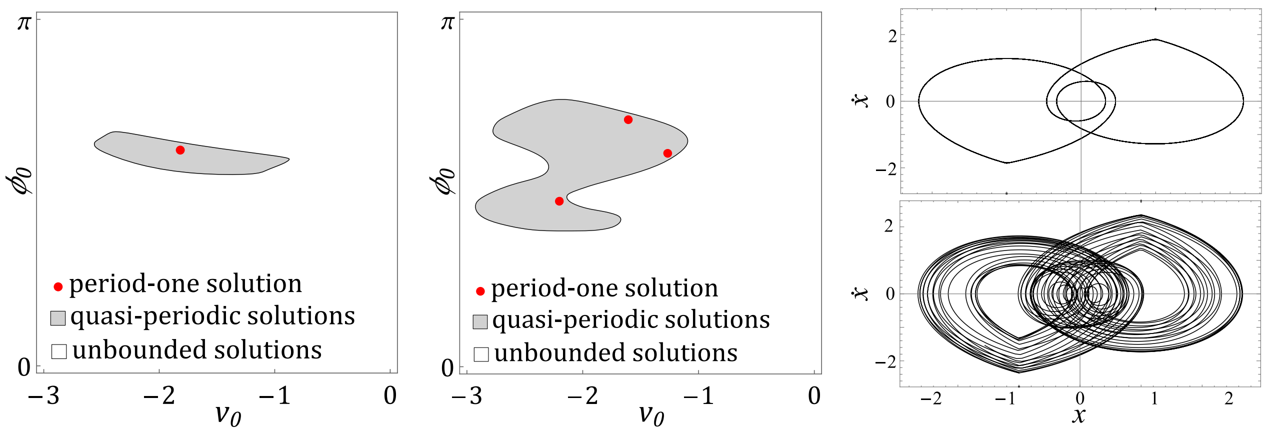
where e is -1 or 1 depending on the initial conditions and the time history of the solution, i.e., whether the solution enters the hysteretic region $-1 < x(t) < 1$ from the left or right. Without loss of generality, we specify the initial conditions as $x(0) = -1$, $\dot{x}(0) = v_0 < 0$, $e = -1$.

Results

We derived the following expressions for the initial conditions of symmetric period-one solutions

$$\cos(\phi_0) = A/(\omega^2 - 1), \qquad v_0 = \omega\tan(\phi_0) - \tan\left(\pi/(2\omega)\right). \tag{3}$$

Varying the forcing parameters we observed coexisting symmetric and asymmetric periodic solutions. Figure 1 illustrates the regions of initial conditions (v_0, ϕ_0) leading to period-one (dots), quasi-periodic (filled regions) and unbounded solutions.



(a) Forcing parameters: $A = 2, \omega = 0.4$ (b) Forcing parameters: $A = 2, \omega = 0.6$ (c) Period-one, quasi-periodic solutions

Figure 1: (a,b) Regions of period-one, quasi-periodic solutions, (c) typical trajectories

Acknowledgement

This project was supported by the by the ÚNKP-19-3 New National Excellence Program of the Ministry for Innovation and Technology of Hungary.

References

[1] Krasnosel'skii, M. A., & Pokrovskii, A. V. (2012) Systems with hysteresis. *Springer, Berlin*.

[2] Krasnosel'skii, A., & Rachinskii, D. (2005) Bifurcation of forced periodic oscillations for equations with Preisach hysteresis. *J. Phys.: Conf. Ser.* **22**(1), 93-102. 5

[3] Kalmár-Nagy, T., Wahi, P., & Halder, A. (2011) Dynamics of a hysteretic relay oscillator with periodic forcing. *SIAM Journal on Applied Dynamical Systems* **10**(2), 403-422.