

Investigations on the nonlinear forced responses of nanobeams modeled with the general nonlocal theory

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Abstract. In this research effort, the linear and nonlinear dynamic responses of nanobeams under forced harmonic excitation are investigated and discussed. For the first time in the literature, the forced vibrations of a nanobeam structure with hinged-hinged boundary conditions modeled with the general nonlocal theory, a modified version of Eringen's nonlocal theory, are studied. The general nonlocal theory is utilized to account for small scale effects, such as long-range interatomic interactions. In expanding the material limits of applicability from Eringen's theory, the general nonlocal theory also increases the complexity of the governing equations of motion and boundary conditions for beam and plate structures. For end constrained Euler-Bernoulli nanobeams with the geometric nonlinearity, the governing equations increase from fourth- to sixth-order and consequently require two additional boundary conditions. The effects that the two size dependent parameters and higher boundary conditions have on the linear and nonlinear dynamic response of a nanobeam subject to harmonic forced excitation are deeply studied in this effort.

Introduction

The development of reduced-order models to study the static and dynamic linear and nonlinear dynamic response of simple nanostructures, such as beams and plates, has grown significantly in recent years [1]. In the development of models for nanostructures, there comes a point when theories in classical continuum mechanics no longer accurately represent the system's material characteristics or phenomena that occur when the internal length scales become comparable to the scale of the considered structure. Essentially, approaching the nanoscale means that one can no longer represent particles in a medium as point masses subject only to translation. Rather, nanovolume representations with additional degrees of freedom are more appropriate [2]. There are several theories aimed at including these additional degrees of freedom. One such theory is Eringen's nonlocal elasticity theory. Eringen's theory includes a single size-dependent parameter that is found by fitting the transverse and longitudinal acoustic dispersion curves and assumes that the stress at one point in a medium is influenced by the strain of neighboring points. This theory has been widely utilized to study the static and dynamic response of nanobeams. Following Eringen's theory, the general nonlocal theory was found. Contrary to Eringen's nonlocal theory, the general nonlocal theory considers two separate attenuation functions for each of the Lamé parameters, thus leading to two size dependent parameters. As such, the general nonlocal theory can simultaneously fit the dispersion curves for certain materials that Eringen's theory cannot. In this modification, the governing equations for a hinged-hinged nanobeam with forced harmonic excitation and the von Karman geometric nonlinearity, shown in Figure 1 increase from fourth to sixth order. Additionally, the two additional boundary conditions that are needed when using the general nonlocal theory are found using the weighted residual approach.



Figure 1: Forced nanobeam with hinged-hinged boundary conditions.

Results and Discussion

Preliminary results for the linear static and dynamic response of an unloaded hinged-hinged nanobeam were recently found [3]. It was shown that when the two size dependent parameters of the general nonlocal theory and the one size dependent parameter of Eringen's theory are equal, the obtained dispersion curves are identical. Additionally, for hinged-hinged boundary conditions and equal nonlocal parameters, there exists a set of two higher-order boundary conditions obtained through a weighted residual approach that yield the exact same linear natural frequencies and mode shapes as Eringen's. As this work progresses, the nonlinear response of the harmonic forced system will be further studied to compare between the two size dependent theories.

References

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