Slipping-rolling transitions of a body with two contact points

Mate Antali* and Gabor Stepan*

* *Department of Applied Mechanics, Budapest University of Technology and Economics, Budapest, Hungary*

Abstract. We analyse a rigid body subjected to dry friction forces, which are acting on the body at two contact points. According to the rolling or slipping at each contact point, the body has four kinematic states. The nonsmooth behaviour of the Coulomb friction law creates two intersecting codimension–2 discontinuity surfaces in the phase space of the system. The analysis of the nonsmooth dynamics at these surfaces determines the qualitative behaviour of the system at rolling-slipping transitions. The interesting feature of the system is the partial indeterminacy of the contact forces at the rolling-rolling case. Even in this situation, the conditions of rolling and slipping can be determined by the appropriate analysis of the nonsmooth system.

Introduction

Consider a rigid body which is connected to fixed rigid surfaces at the contact points P^- and P^+. We assume that the problem is three-dimensional in the sense that in slipping state, the velocities $\mathbf{v}^-$ and $\mathbf{v}^+$ of the contact points can have two components. Some appearances of this scenario in mechanical engineering examples can be seen in Figure 1. Let us assume that the normal contact at both points is continuously ensured, and there is dry friction at the contact points described by the Coulomb friction model. Then, the body has four kinematic states according to the dynamic (sticking) or static (rolling or sticking) state of the contact points. Under the effect of external loads, the body may switch between these states. These switches were analysed in [4] in the case of a single contact point. The interesting dynamics of these systems originates from the fact that in the two-point contact case, the constraints $\mathbf{v}^- = \mathbf{0}$ and $\mathbf{v}^+ = \mathbf{0}$ are not independent.

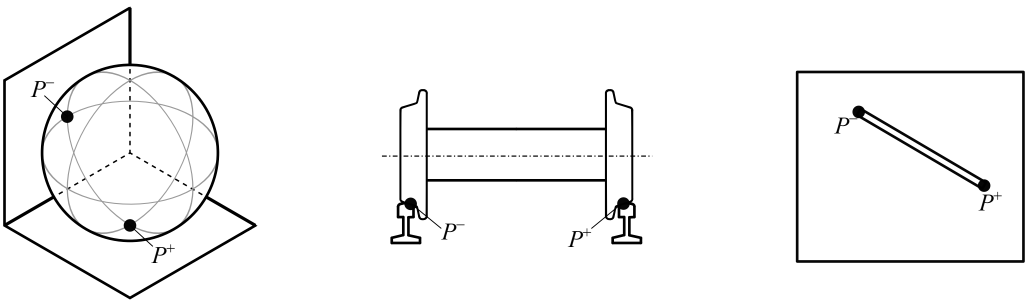

Figure 1: Different geometric realizations of the two-point contact problem. Left panel: a ball connected to two planar surfaces, which is an approximate model of a rotating ball flowmeter [1]. Middle panel: a railway wheelset rolling on the two rails [2]. Right panel: two mass points connected by a rigid bar and moving on a rough surface, which can be a compressed member of a tensegrity robot structure [3].

Results and discussion

The normal contact conditions are two holonomic constraints, thus, the state of the body can be described locally by four generalized coordinates $q = (q_1, q_2, q_3, q_4)$ and quasi-velocities $s = (s_1, s_2, s_3, s_4)$. When both contact poins are in slipping state, the dynamics of the body is given in the form $\dot{x} = f(x)$ in the state space of $x = (q, s)$. The conditions $\mathbf{v}^+(q, s) = \mathbf{0}$ and $\mathbf{v}^-(q, s) = \mathbf{0}$ determine codimension–2 discontinuity sets of the system, and they correspond to the mixed static-dynamic states of the body. The rolling-rolling (or sticking-sticking) state of the body is located at the intersection of these surfaces. By analysing the nonsmooth vector field, we can determine the possible directions of slipping velocities where rolling-slipping transitions can occur. The attracting or repelling behaviour of the vector field along these directions determine the qualitative behaviour of the system near the static states. In addition, this analysis provides an alternative method for checking the fulfilment of the rolling conditions. Moreover, we can get results even for the case when the contact forces are not completely determined due to the redundancy of the rolling constraints at the two contact points.

Acknowledgement The research work of the first author is supported by the Hungarian Academy of Sciences in the Premium Postdoctoral Fellowship Programme under the grant number PPD2018-014/2018.

References

[1] Antali M., Stepan G. (2016) Discontinuity-induced bifurcations of a dual-point contact ball. *Nonlinear Dynamics* **83**(1):685–702.

[2] Antali M., Stepan G. (2016) On the nonlinear kinematic oscillations of railway wheelsets. *Journal of Computational and Nonlinear Dynamics* **11**(5):051020.

[3] Schorr P., Bhm V., Zentner L., Zimmermann K. (2020) Design of a vibration driven motion system based on a multistable tensegrity structure. In: Gusikhin O., Madani K. (eds) *Informatics in Control, Automation and Robotics. ICINCO 2018.*

[4] Antali M., Stepan G. (2019) Nonsmooth analysis of three-dimensional slipping and rolling in the presence of dry friction. *Nonlinear Dynamics* **97**(3):1799–1817.