Statistical analyses of an iterative algorithm class for dynamical systems

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Abstract. We present statistical analyses of algorithms to compute complete Lyapunov functions for dynamical systems. The algorithms construct a complete Lyapunov function by approximately solving an equation for the orbital derivative using mesh-free collocation with radial basis functions over a collocation grid. The equation has no solution in the chain-recurrent set, and hence we use points with poor approximation to determine the chain-recurrent set. To judge whether the approximation is poor, we evaluate the orbital derivative on an evaluation grid, corresponding to each collocation point. Two types of evaluation grids are used: either we place evaluation points in a circle (or sphere) around each collocation point, or we place them in the direction of the flow around each collocation point. Further, in an iterative algorithm we use the average of the orbital derivative on the evaluation grids to re-iterate. In this paper we deal with two open questions: which of the two types of evaluation grids gives better results? Is the average the adequate quantity or is the median more effective?

Introduction

Let us consider $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$, where $\mathbf{x} \in \mathbb{R}^n$, $n \in \mathbb{N}$. A *Lyapunov function* for that system is an auxiliary scalar-valued function, whose domain is a subset of the state-space and which is (strictly) decreasing along all solution trajectories in a neighbourhood of an attractor, such as an equilibrium point or a periodic orbit. A natural extension is a function defined on the whole state space, a *complete Lyapunov function* (CLF), see [1]. A CLF allows for dividing the state-space into two disjoint areas with gradient-like flow, where the solution trajectories flow through, and the chain-recurrent set, where infinitesimal perturbations can make the system recurrent. On the gradient-like part, a CLF V is strictly decreasing along solutions, whereas it is constant on the chain-recurrent set. This can be expressed by the orbital derivative $V'(\mathbf{x}) = \nabla V(\mathbf{x}) \cdot \mathbf{f}(\mathbf{x})$, i.e. the derivative along solutions, being strictly negative or zero, respectively [2]. The general idea is to find a CLF with $v'(\mathbf{x}) \leq 0$ by approximating a "solution" to the ill-posed problem $V'(\mathbf{x}) = -1$. If the approximation is unsatisfactory, it can be enhanced with an iteration procedure. To evaluate the CLF we use two different grids, Figure 1. This paper studies the influence of the distribution of the points in the evaluation grids for the reconstruction of a CLF.

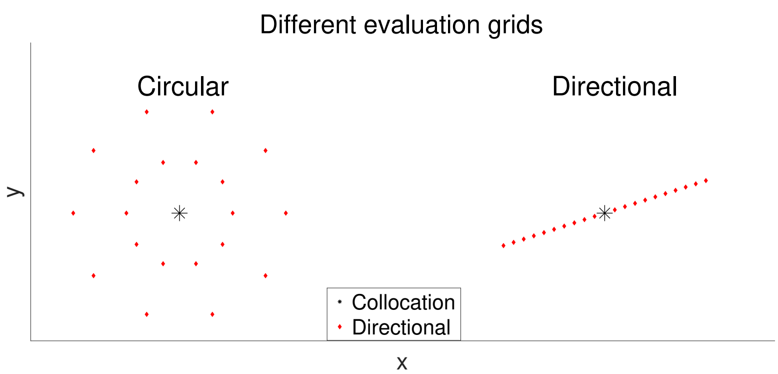

Figure 1: Circular directional grid points around collocation points.

Results and discussion

The directional evaluation grid provides better results than the circular evaluation grid. An explanation for this is that the directional evaluation grid only considers points along the flow, which are of the same type as the collocation point. In particular, if the collocation point is on (or very close to) the chain-recurrent set, then so are the evaluation points and this helps determining that the collocation point belongs to the chain recurrent set.

References

[1] Conley. C. (1978) Isolated Invariant Sets and the Morse Index. *CBMS Regional Conference Series* no. **38**. American Mathematical Society.

[2] Argáez, C. and Giesl, P. and Hafstein, S. (2018) *Iterative construction of complete Lyapunov functions* Proceedings of the 8th International Conference on Simulation and Modeling Methodologies, Technologies and Applications (SIMULTECH), Porto, Portugal 211-222.