An approximate approach for solving the steady-state response of a two-stage nonlinear vibration isolation system

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Abstract. The steady-state response of a two-stage vibration isolation system with geometrically nonlinear stiffness and damping is obtained by Jacobi elliptic functions. The harmonic components of stiffness and damping force in the harmonic balance method are modified based on the orthogonality relationship of force components. The results show that the amplitude-frequency response curves obtained using the current approach can be compared well with the numerical results.

Introduction

A two-stage vibration isolation system is efficient in isolating mechanical equipment. To improve the isolation characteristics, researchers have investigated the effects of the nonlinear stiffness and damping elements in the system. Figure 1 is a lumped parameter model of a two-stage nonlinear vibration isolator with geometrically nonlinear stiffness and damping [1]. The harmonic balance method (HBM) can be applied to determine the steady-state response of the system. However, when the high order harmonic components are neglected, there is inevitably truncation error. This paper aims to improve the accuracy of the steady-state response of the two-stage nonlinear vibration isolator using Jacobi elliptic functions [2-3]. The non-dimensional equation of motion of the system in Fig. 1 is approximately given by

$$\hat{\mathbf{M}}\hat{\mathbf{x}}'' + \hat{\mathbf{C}}_{\mathbf{n}}\hat{\mathbf{x}}' + \hat{\mathbf{C}}_{\mathbf{v}}\hat{\mathbf{x}}' + \hat{\mathbf{K}}_{\mathbf{1}}\hat{\mathbf{x}} + \mathbf{K}_{\mathbf{3}}\hat{\mathbf{x}}^{(3)} = \hat{\mathbf{f}}\,, \tag{2}$$

where the nondimensional parameters are similar to those given in Ref. [1]. The relative displacement between m_{s1} and m_{s2} is $\hat{x}_r(\tau) = \hat{x}_1(\tau) - \hat{x}_2(\tau)$. When the relative displacement between m_{s1} and m_{s2} and displacement of m_{s1} are assumed by using Jacobi elliptic functions $\hat{x}_r(\tau) = \hat{X}_r \mathrm{cn}(\psi_r | m_r)$ and $\hat{x}_2(\tau) = \hat{X}_2 \mathrm{cn}(\psi_2 | m_2)$ respectively, the elliptic harmonic balance method (EHBM) [3] can be used to modify the harmonic components of stiffness and damping force in the HBM based on the orthogonality relationship of force components.

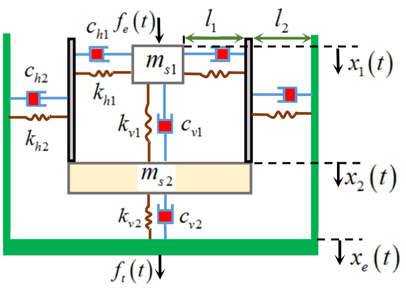

Figure 1: Model of a two-stage nonlinear vibration isolator.

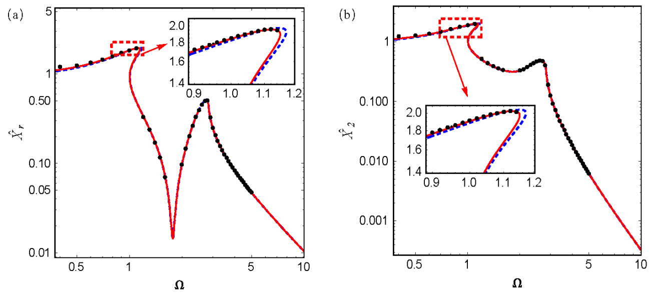

Figure 2: Amplitude-frequency curves of (a) the relative displacement between m_{s1} and m_{s2}, (b) m_{s2}.

Results and discussion

The amplitude-frequency curves of the relative displacement between m_{s1} and m_{s2}, and m_{s2} are determined using the EHBM and shown in Fig. 2 (solid line). Also plotted are the results obtained using the HBM (dashed line) and the fourth-order Runge-Kutta method (dotted line). It can be seen that the amplitude-frequency response curves obtained by using the EHBM can be compared well with the numerical results. The EHBM results are better than those of the HBM around the first peak regime, where the HBM induces the truncation error by neglecting the high order harmonic components. The EHBM can also be used to obtain the force and displacement transmissibility of the two-stage nonlinear vibration isolator with geometric stiffness and damping. (Acknowledgment: NSFC Grant No. 11672058)

References

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