Multijump resonance in a class of oscillators with nonic polynomial nonlinearity

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Abstract. In this paper the design of a multijump resonance system with a nonic nonlinearity is explored. Multijump resonance is a phenomenon observable in driven nonlinear systems which leads to a multi-valued frequency response, showing several hysteresis windows with respect to the frequency of the driving signal. The possibility of exploiting this peculiar behavior for the realization of an analog memory device is explored.

Introduction

The phenomenon of jump resonance is a fundamental topic in the area of nonlinear control. It identifies an hysteretic behavior in the frequency response of forced nonlinear systems involving the presence of sudden jumps in the amplitude of the output signal for particular values of the frequency of the input signal [1]. Since jump resonance may represent a desirable "memory" effect, it is necessary a strategy to design electronic circuits with multijump resonance [2]. Furthermore, with the term "multijump" resonance, we indicate a frequency response where more frequency hysteresis windows are present within the same range, thus leading to multiple jump paths. In this contribution we highlight the occurrence of a complex pattern of multijumps in a nonlinear oscillator involving a nonic nonlinearity.

Results and discussion

Given a generic second-order linear system with transfer function $G(s) = \frac{K\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$, it can be put in closed loop scheme, better known as Lur'e form, where $G(s)$ is the transfer function of the linear part and $\phi(X)$ represents the describing function of the nonlinearity $\psi(x)$. Assuming $\psi(x) = ax^9 + bx^5 + cx^3 + dx$, being an odd symmetric polynomial, we can use the following single-input describing function (SIDF) $\phi(X) = AX^8 + BX^6 + CX^4 + DX^2$ where $A = \frac{9}{32}a$, $B = \frac{7}{16}b$, $C = \frac{5}{8}c$ and $D = \frac{3}{4}d$ are coefficient related to the describing function of odd symmetric polynomials.
Taking into consideration the Lur'e scheme, the following equation, representing the closed-loop response for $s = j\omega$, is calculated as $\left[\phi(X) + G^{-1}(j\omega)\right] X = r$. Assuming $G^{-1}(j\omega) = R(\omega) + jI(\omega)$ and calculating the modulus we have:

$$[X\phi(X) + RX]^2 + I^2 = r^2 \tag{1}$$

A polynomial of eighteenth order with only even coefficients in X is thus obtained. When this polynomial admits 9 positive and real solutions, the closed loop system shows multijump resonance. In particular, by opportunely searching the coefficients of the nonlinearity ensuring this condition, it is possible to select $a = 1$, $b = -38.09$, $c = 499.68$, $d = -2569$, obtaining the frequency response showing a nine windows multijump region, reported in Fig. 1.

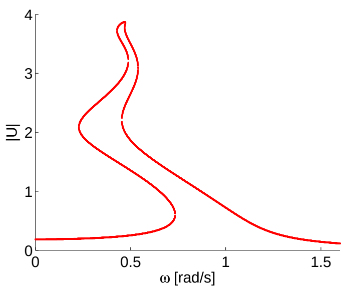

Figure 1: Multijump resonance with nine windows of hysteresis in the nonic nonlinear oscillator. Parameters are selected as: $a = 906157$, $b = -38.09$, $c = 499.68$, $d = -2569$, $K = 0.00019$, $\xi = 0.05$, $\omega_n = 1$. The input is a sinusoidal signal with amplitude $r = 950$ and frequency ω.

Indeed both the choice of the parameters of the linear part, those of the nonlinearity and the amplitude of the forcing signal assures us the existence of family of curves with the same peculiarity as that shown in Fig. 1. Therefore the possibility of using a simple nonlinear oscillator also as multi-state memory make further impressive the capabilities of this simple device.

References

[1] Atherton, D.P. (1982) *Nonlinear control engineering*, (Van Nostrand Reinhold Co. Ltd., Wokingham, UK).
[2] Buscarino, A., Famoso, C., Fortuna, L. and Frasca, M., 2020. Multi-jump resonance systems. International Journal of Control, 93(2), pp.282-292.