On the effects of added devices on the nonlinear behaviour of the damped Beck's beam

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Abstract. The effects of added devices on the nonlinear behaviour of the damped Beck's beam are discussed in this work. The system is modelled as an inextensible and shear-undeformable cantilever beam, with distributed damping, loaded at the free end by a follower force. It is also equipped with lumped devices, namely linear and/or nonlinear springs and dashpots, placed at generic positions along the beam axis. Stability analysis of the trivial rectilinear configuration is performed and the Hopf bifurcation conditions are detected. A multiple-scale approach is directly applied to the integro-differential equations of motion to analyse the effects of added devices on the limit-cycle, occurring once the Hopf critical load is overcome.

Introduction

The Ziegler Paradox is a well-known mechanical phenomenon, according to which, when a positive definite and vanishing damping is added to a circulatory system, a finite lowering of the Hopf bifurcation load, is experienced [1]. The visco-elastic Beck's beam [2], namely a cantilever beam, with internal and external distributed damping, and loaded at the free end by a follower force, is a continuous system, that is paradigmatic of such a paradox. Its linear behaviour has been widely investigated in the literature also to improve, with added damping devices, the beam's stability [3], i.e. to increase the Hopf critical load. The nonlinear behaviour of the visco-elastic Beck's beam, equipped with lumped devices at the tip, is addressed in [4] to investigate the scenario close to the double-zero bifurcation. In [5] the effects of distributed nonlinear hysteretic damping on the post-critical scenario around the Hopf bifurcation are discussed.
This research work is framed in the scenario illustrated above, and it aims to analyse the nonlinear dynamics of the visco-elastic Beck's beam, equipped with linear and/or nonlinear spring/dashpot devices, close to the Hopf bifurcation. Both the effects of the choice of different devices and of their location along the beam axis are discussed. To this end an asymptotic approach, grounded on the Multiple Scale Method, is developed. The obtained bifurcation equation, governing the dynamics close to the Hopf bifurcation, is analysed; moreover, the semi-analytical results are compared with those obtained through a numerical approach, directly applied on the discretized integro-differential equations of motion.

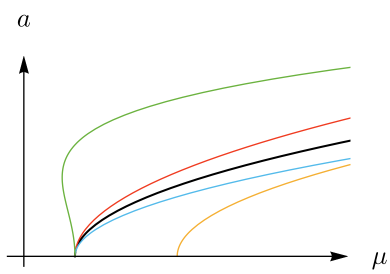

Figure 1: Sketch of the bifurcation diagram, amplitude a vs load μ, of the visco-elastic Beck's beam, with different added devices.

Results and discussion

The main findings of this work can be summarized in: (i) the choice of the constitutive law of the added devices has a strong influence on the critical and post-critical scenario of the Hopf bifurcation, as sketched in Fig. 1, where it is shown as the bifurcation diagram of the visco-elastic Beck's beam (black curve) can be shifted and/or deformed by added devices (green, red, cyan and yellow curves); (ii) the proper location of the added devices is a crucial task in system's design since it may induce beneficial or detrimental effects on its response; (iii) an overall good accordance between the asymptotic results and benchmark numerical solutions has been observed, thus entailing that the developed multiple-scale algorithm is a reliable tool to predict the beam's nonlinear behaviour.
Finally, it is believed that this work can represent a first step towards the challenging task of a proper design of lumped devices, targeted to improve the performances of the visco-elastic Beck's beam.

References

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