Nonlinear dynamics of spherical caps

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Abstract. In the present work the nonlinear vibrations of shallow spherical caps, under the action of static and fluctuating pressure are studied. A semi-analytical approach, based on the Novozhilov's nonlinear thin shell theory, is developed; the approach is suitable for treating homogeneous isotropic shells. A meshless method is considered to reduce the partial differential equations (PDEs) to a set of ordinary differential equations (ODEs): the displacement fields are expanded though a mixed series of Legendre polynomials and harmonic functions in the azimuthal and circumferential directions respectively. The ODEs are obtained by taking advantage from the Lagrange equations and are numerically analysed using continuation techniques, more specifically: static bifurcations of equilibrium points; stability and bifurcations of periodic orbits. The achievements of this study show that nonlinear modal interactions can lead to the activation of non-symmetric vibrational states.

Introduction

The present study is focused on the nonlinear vibrations of spherical thin walled caps, which are a kind of structure widely used in engineering: pressure vessels, aerospace and aeronautical components, civil structures like roofs.

In this study a new method is proposed to analyse axisymmetric and asymmetric vibrations of thin walled shallow spherical caps under the action of uniform static and fluctuating pressure; the aim is investigating possible nonlinear modal interactions that can to the activation of asymmetric modes.

The most of previous theoretical studies on this topic were focused on axisymmetric vibrations [1,2], which neglect the possible onset of non-symmetric vibrations due to the activation of asymmetric modes via nonlinear interactions.

In order to develop the method, the Novozhilov's thin shell theory is considered [3], such theory allow an accurate kinematical modelling of large thin shell deformations. In order to analyse the nonlinear PDE set arising from the Novozhilov theory a meshless discretization approach is considered: the three displacement fields are expanded through a double series, in the azimuthal direction (Figure 1a) Legendre polynomials are considered and combined in order to respect boundary conditions, in the circumferential direction (Figure 1b) a Fourier series is used due to the periodicity; figures 1a,b show the shell geometry as well as the three displacement fields u, v, w, the azimuthal variable φ and the circumferential variable ϑ.

The main results of this study are summarized in Figure 1c, where the amplitude of the modal coordinate of the axisymmetric mode (1,0), referred to the transversal displacement field w, is represented v.s. the normalized excitation frequency of the fluctuating part of the pressure. One ca see that the axisymmetric oscillation loses stability close to the resonance, the instability of axisymmetric vibration gives rise to the onset of asymmetric vibration of the spherical cup, even though the uniform pressure provides axisymmetric excitation

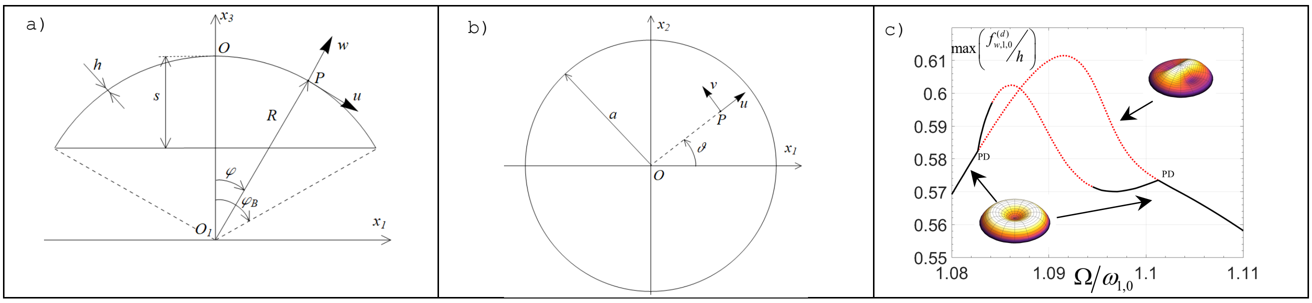

Figure 1: Geometry of the cap, a) lateral view, b) top view. Amplitude frequency diagram c).

References

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