Physically informed data-based closure schemes for turbulent systems

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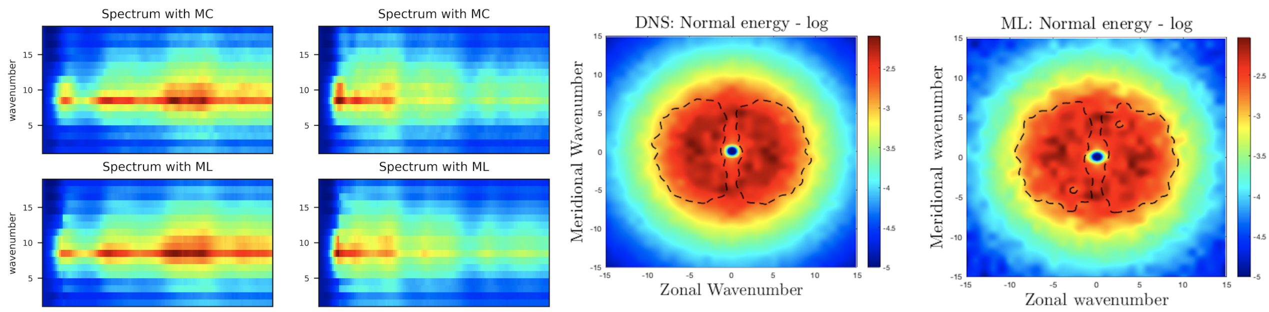
Abstract. We explore the capacity of deep recurrent neural networks to act as nonlocal closure models for a number of different turbulent systems with focus in fluid dynamics. We develop closures for the mean field, as well as for the covariance of perturbations. The generic form of the systems we study includes a linear part, external forcing and a quadratic and energy preserving term. In all cases, we employ a coarse-discretization of the governing equations together with the neural net closures to evolve the fluid flow in time. We utilize recurrent and convolutional layers to capture both temporal and spatial non-local effects respectively. Furthermore, we train our models by imposing physical constraints regarding potential energy-preserving of the nonlinear closure terms. Our numerical investigations include problems of practical interest such as bubble-fluid multiphase flows and turbulent high-latitude quasi-geostrophic flows.

Introduction

We study turbulent systems that include an energy-preserving quadratic operator and external forcing with a mean component and a stochastic fluctuations with white-noise characteristics. As regularly done in uncertainty quantification (UQ), e.g. [1], we model the state of the system by its mean and a fixed orthonormal basis. We can then obtain the equation for the covariance matrix and mean state of the system. To compute the statistics of such a system, we incorporate machine learning (ML), in the form of recurrent deep neural networks and spatio-temporal convolutions to device a closure scheme. Similar data-based approaches have been shown to outperform analytical moment-closure assumptions in low-dimensional systems [2]. In the case of turbulent systems, however, energy constraints are extremely important to ensure that the closure assumptions don't lead to numerical instabilities [3]. We impose such energy constraints, arising from the nature of the quadratic operator, during the training of our neural networks, to improve the stability of our model.



(a) (b)

Figure 1: Comparison of our data-based UQ approach and the Monte-Carlo method for (a) the Lorentz-96 system for different realizations of the stochastic forcing (b) the energy-spectrum of the statistically steady-state of a high-lattitude QG turbulent flow.

Results and discussion

To showcase the validity of our approach we test our model on the L-96 model, as well as on turbulent quasi-geostrophic flows and multiphase flows in the ocean and atmosphere. Our data-informed closure scheme is then applied on coarse-scale resolutions of the problems and compared to fine-scale direct numerical simulations. The numerical tests showcase both that no numerical instabilities occur, a crucial property in chaotic dynamical systems, but also very good agreement between our ML-enhanced predictions and Monte-Carlo (MC) simulations. Finally, our incorporation of spatial-temporal convolutions, allows for a much faster method compare to the computationally demanding MC runs.

References

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