Negative Potential Energy and Stiffness Content in Accelerated Cracked Rotor System

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Abstract. For transient operation in cracked rotors, the equations of motion of the system are affected by the angular acceleration rate which results in linear-time varying (LTV) equations of motion. Consequently, the Floquet's theory becomes not applicable for stability analysis of such system. Therefore, an effective equivalent stiffness measure is introduced in this study to analyze the effect of the crack depth and the unbalance force vector orientation on the stability of a two-degree-of-freedom cracked system during transient operation. Accordingly, at varying crack depths for a wide range of angular acceleration rates and unbalance force vector angles, the potential energy and its corresponding effective stiffness content have been found to be dominated by negative values. The intensity of these zones is found to be significantly affected by the unbalance force vector orientation and the angular acceleration rates.

Introduction

Several crack detection methods were addressed in the review of state-of-the-art [1] in which different kinds of cracked rotor models, like finite element and Jeffcott rotor models and various methods of whirl response and stability analysis were reviewed and discussed. Accordingly, unstable zones are usually obtained by applying Floquet's theory to the free response of the parametrically excited cracked rotor system where the state-transition matrix is formulated using an identity matrix of initial conditions [2]. The obtained unstable zones from the eigensolution of the state-transition matrix are usually associated with negative potential and stiffness energy content. However, the accelerated rotor systems are considered linear-time-variant (LTV) rather than linear-time periodic systems where Floquet's theory is not applicable. Therefore, an effective stiffness measure is introduced here based on [3] to investigate the effect of propagating crack in accelerated rotor system subjected to unbalance force excitation on negative potential and stiffness contents.

Model and results

The Jeffcott rotor model is considered for the vector of horizontal and vertical whirl amplitudes $\mathbf{q}(t)=[u(t) \quad v(t)]^{\mathrm{T}}$ where the LTV equations of motion are written in matrix form as

$$\mathbf{M}\ddot{\mathbf{q}}(t)+\mathbf{C}\dot{\mathbf{q}}(t)+\mathbf{K}_C(t)\mathbf{q}(t)=\mathbf{F}_u(t) \tag{1}$$

where $\mathbf{K}_C(t)=\mathbf{K}_1+\mathbf{K}_2\cos(2\theta(t))+\mathbf{K}_3\sin(2\theta(t))$, $\theta(t)=\alpha t^2/2$ and α is the angular acceleration rate. The matrices $\mathbf{K}_1$, $\mathbf{K}_2$, and $\mathbf{K}_3$ are found in [3]. From the integration of the equations of motion, the instantaneous potential energy and effective stiffness formulas at the resultant whirl amplitude $z(t)$ are obtained as

$$P(t)=\int_0^t \dot{\mathbf{q}}^T\mathbf{K}_C(t)\mathbf{q}\,dt \;,\; k_{eff}(t)\cong\frac{2P(t)}{z(t)^2} \text{ and } z(t)=\sqrt{u(t)^2+v(t)^2} \tag{2}$$

The obtained potential energy and its corresponding effective stiffness values based on above equations are plotted in Figures 1a and 1b, respectively based on the experimental whirl amplitudes of the setup in Figure 1c. It is observed that in the vicinity of the critical rotational speed $\Omega_{exp}\cong 54.5$ Hz, significant transitions between positive and negative effective stiffness values take place.

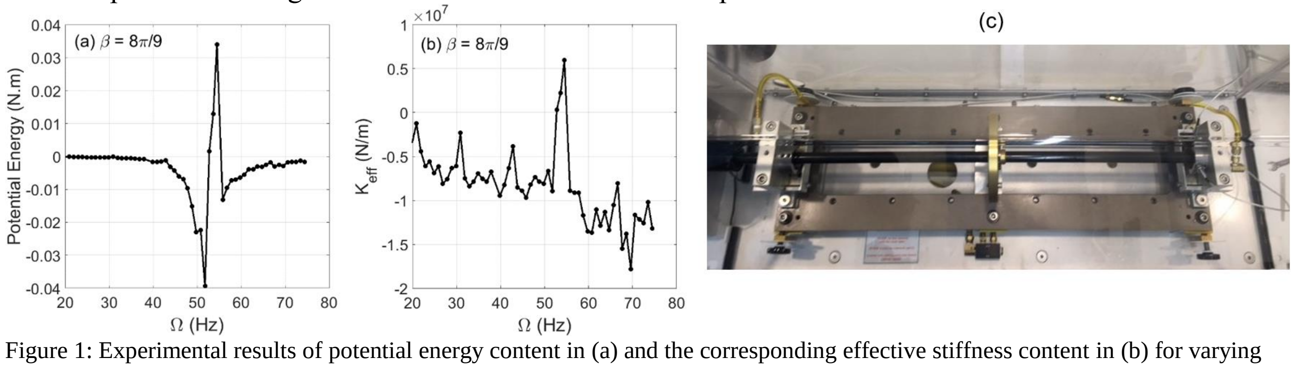

Figure 1: Experimental results of potential energy content in (a) and the corresponding effective stiffness content in (b) for varying shaft rotational speeds at $\mu=0.2$ and $\beta=8\pi/9$ (2.8 rad).

References

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