# Vibrations and buckling characteristics of a carbon nanotube embedded in a viscoelastic foundation

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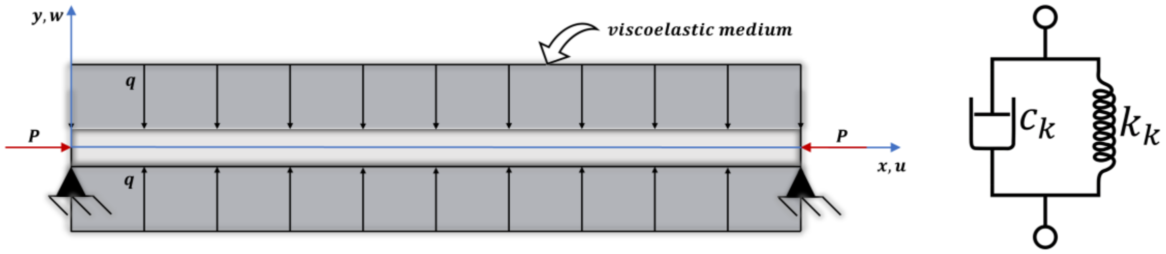
**Abstract**. In this study, the vibrations and buckling characteristics of a hinged-hinged single-walled carbon nanotube (SWCNT) embedded in varying viscoelastic foundations are investigated and discussed. The effects of axial loading, mid-plane stretching, size-dependent phenomenon, and modal damping are considered. The von Kármán geometric nonlinearity is introduced into the system and allows for the determination of the post-buckled static configuration and post-buckled natural frequencies. Using Eringen's nonlocal elasticity theory and by virtue of the extended Hamilton's principle, the nonlocal transverse governing equation for the SWCNT is derived. The dependence of the nonlocal parameter and viscoelastic coefficients for Kelvin-Voight, Maxwell, Standard Solid I and II, and Standard Fluid I and II on the critical buckling load and natural frequencies are presented.

## Introduction

The study of different types of surrounding viscoelastic media and their effects as surrounding foundation on the static and dynamic response of simple nanostructures can prove useful in a variety of applications, including targeted drug delivery. As previously seen in [1, 2], a mathematical model for an elastic SWCNT may be developed using Euler-Bernoulli beam theory, the nonlinear von Kármán strain tensor, and Eringen's nonlocal elasticity theory. Using the variational kinetic and potential energies, the general nonlocal equation of motion (EOM) can be derived by virtue of Hamilton's principle and made dimensionless. Additionally, in the derivation of the governing equations, the viscoelastic foundation is represented by a distributed load $q$ acting along the length of the nanobeam. In each viscoelastic medium, there exists elastic and viscous elements in parallel, series, or a combination of the two, depending on the viscoelastic foundation. The configuration of the elastic and viscous elements can significantly influence the static and dynamic response of the axially loaded nanobeam. The proposed system and one of the viscoelastic representations (Kelvin-Voight) are shown in Figure (1), following [3].




(a) (b)

Figure 1: (a) Forced nanobeam with hinged-hinged boundary conditions with surrounding viscoelastic foundation and (b)Kelvin-Voight representation.

## Results and Discussion

The critical buckling load, static post-buckled configurations, and pre-/post-buckled natural frequencies and mode shapes are determined for different nonlocal parameters and viscoelastic foundation elastic and viscous coefficients. It was found that the Kelvin-Voight, Standard Solid I, and Standard Solid II delayed the occurrence of buckling for increase elastic coefficients. The Maxwell, Standard Fluid I, and Standard Fluid II models did not alter the critical buckling values due to the elastic and viscous elements being in series. In all cases, increasing the nonlocal parameter decreased the critical buckling values. For the dynamic analysis, it was shown that increasing the axial load and nonlocal parameter in pre-buckling lowers the first natural frequency. Additionally, trends on increasing and decreasing natural frequencies for the different two and three element viscoelastic foundation were explained by the elements acting either in series or in parallel.

## References

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