Piezoelectric control of the Hopf bifurcation of the visco-elastic Beck's beam

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Abstract. The critical and post-critical scenario of the visco-elastic Beck's beam, endowed with a piezoelectric controller, are here presented. The model of the Piezo-Electro-Mechanical (PEM) system consists of an inextensible and shear-undeformable visco-elastic cantilever beam, loaded at the free end by a follower force, that is coupled to a layer of distributed piezoelectric devices, shunted to a nonsimilar electric circuit. The equations of motion are derived in finite kinematics and a linear stability analysis is carried out to detect the parameters combinations at which Hopf bifurcation occurs. Then a multiple-scale approach is adopted to investigate the post-critical behavior of the PEM system, i.e. the amplitude of the limit-cycle occurring once the Hopf critical load is overcome.

Introduction

The destabilizing effect of damping, or the 'Ziegler paradox', has been widely investigated in the literature, i.e. the introduction of a small positive damping may cause a finite reduction in the critical load of the Hopf bifurcation in circulatory systems. The paradigmatic continuous mechanical system resembling the paradox is the Beck's beam [1], which inspired several efforts to study the phenomenon in the linear field, i.e. to better understand the fashinating paradox [2] and to, possibly, improve, e.g. with added piezoelectric devices, the Beck's beam stability [3]. However, few works have been proposed to study the effect of the Ziegler paradox in the post-critical behavior of nonlinear systems [4].

Here, a PEM system is investigated, see Fig. 1-a: the beam (gray element) of length ℓ, is clamped in A and is subject to the follower force μ in B; two (idealized as continuous) layers of piezoelectric patches (light blue elements) are attached at the top and the bottom of the beam and are shunted to a zero-order network and zero-order analog electric circuit (E.C.). The governing equations are derived in finite kinematics and linear stability

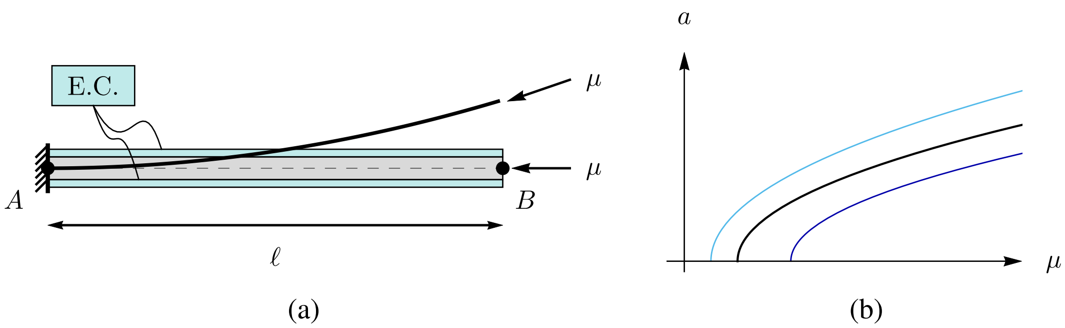

Figure 1: The PEM system (a) and a sketch of its bifurcation diagram in terms of amplitude a vs load μ (b).

of the trivial rectilinear configuration is analyzed to detect the Hopf bifurcation conditions. Then a multiple-scale procedure is developed in direct form to derive the bifurcation equations. The nonlinear response of the PEM system is then systematically compared to that obtained via a direct numerical approach. Finally, a parametric analysis is conducted to investigate the effect of the electric parameters and verify the possibility to reduce the limit-cycle amplitude.

Results and Discussion

The results obtained in this research work are sketched in Fig. 1-b and can be summarized as follows: (i) the adopted piezoelectric controller is capable of introducing beneficial effects in the PEM system nonlinear response (dark blue line) compared to that of the uncontrolled beam (black line), though detrimental effects may be introduced if the controller is not properly designed (light blue line); (ii) the parametric analyses delivered a suitable parameter space in which the Hopf critical load is enhanced and the limit-cycle amplitudes are significantly reduced; (iii) the systematic comparison with the numerical solution, revealed that the proposed perturbation procedure allows to well predict the nonlinear dynamics of the PEM system.

References

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