# Effects of strong viscosity with variable fluid properties on falling film instability

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**Abstract**. A gravity-driven thin liquid film flow on a heated, inclined plane is considered within the framework of the long-wave approximation method. A nonlinear evolution equation for the film thickness is derived through a long-wave theory. We perform a linear stability analysis and obtain the critical Reynolds number. Applying the method of multiple scales we demarcated different instability zones. The study reveals that $K_{\rho}$, $K_{\mu}$ and $K_{\sigma}$ has substantial effects on different stability zones. We have investigated numerically the nonlinear evolution equation in a periodic domain and found substantial effects of $K_{\rho}$, $K_{\mu}$ and $K_{\sigma}$.

## Introduction

The effect of variable thermophysical properties with temperature for a thin film flow along an inclined rigid plane is a challenging problem due to its complicated formulation and complexity of the problem. To avoid this complexity most of the authors considered the thermophysical properties, such as viscosity, density, thermal conductivity, surface tension, etc. as constants. Goussis and Kelly [1] first studied the effects of viscosity variation on the stability of a non-isothermal thin liquid film flow on a rigid inclined plane where they assumed the variation of the viscosity to be exponential with the temperature which is an approximation to the Arrhenius-type relation. Pascal et al. [2] first studied the instability of a thin film along a uniformly heated incline, where they considered the linear variation of different fluid properties such as density, thermal conductivity, viscosity and specific heat and surface tension. Mukhopadhyay and Chattopadhyay [3] first proposed an analytical model for long-wave instability of thin film flowing down a uniformly heated inclined plane, considering the linear variation of various thermophysical properties where they presented both linear as well as weakly nonlinear stability analysis. In the current study, we consider the variation of viscosity to be exponential that is Arrhenius-type relation. The other thermophysical quantities such as density, surface tension, and thermal conductivity are assumed to vary linearly with the moderate variation of temperature.

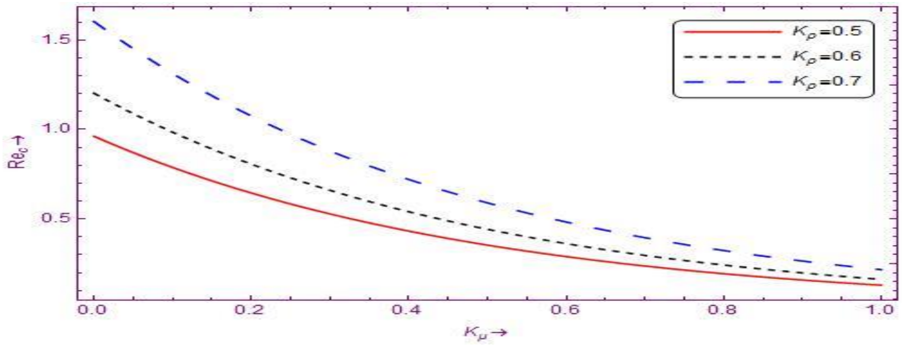


Figure 1: Variation of critical Reynolds number with the variation of $K_{\mu}$ for different $K_{\rho}$ with fixed $\gamma = \pi/3$.

## Formulation of the problem

We consider a two-dimensional gravity-driven incompressible viscous thin liquid film flow along an heated inclined plane with an angle of inclination $\gamma$. The interfacial surface of the wavy thin film may be expressed as $z = h(x, t)$, where $h$ denotes the thickness of the film at time $t$. Finally, it is assumed that, for moderate temperature variation, the density, surface tension, and thermal diffusivity vary linearly with temperature and viscosity varies exponentially. Here $K_{\rho}$, $K_{\mu}$ and $K_{\sigma}$ are the parameters measuring the rate of change of the corresponding quantity for the variation of temperature.

## Key Results

1. $K_{\rho}$, $K_{\mu}$ and $K_{\sigma}$ have significant effects on the flow filed.
2. Increment of $K_{\rho}$ stabilizes the flow whereas $K_{\mu}$ and $K_{\sigma}$ destabilizes the flow.
3. Different instability zones highly influenced by these parameters.
4. Threshold amplitudes and nonlinear wave speed highly depends on these factors.

## References

[1] Goussis D.A, Kelley R.E. (1985) Effects of viscosity variation on the stability of film flow down heated or cooled inclined surfaces: Long-wavelength analysis. Physics of Fluids 28:3207-3214.

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