Forward sensitivity analysis of the FitzHugh-Nagumo system

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Abstract. The FitzHugh-Nagumo model, from computational neuroscience, has attracted attention in nonlinear dynamics studies since it exhibits interesting bifurcation characteristics. The estimation of the model parameters and their time dependence accurately is vital to understand how the model evolves in time. To this end, we provide a forward sensitivity method (FSM) approach to quantify the main model parameters using a few measurement data. FSM constitutes a variational data assimilation technique which integrates model sensitivities into the process of fitting the model to the observation in least squares sense. We analyse the applicability of FSM to update the FitzHugh-Nagumo model parameters and predict its dynamical behavior. We also explore its performance using various sets of observational operators, sparsity, and noise levels.

Introduction

The two-equation FitzHugh-Nagumo (F-N) model describing neuronal spike discharges can be defined as

$$\frac{\mathrm{d}V}{\mathrm{d}t} = V - \frac{1}{3}V^3 - W + I, \tag{1}$$

$$\tau\frac{\mathrm{d}W}{\mathrm{d}t} = V + a - bW, \tag{2}$$

where V is the membrane potential, W is a recovery variable, and τ is the time scale. I represents the external input current, while a and b are the controlling parameters of the model. The fixed point of the F-N model correspond to setting the right hand side of Eq. 1-2 to zero, i.e., $V - \frac{1}{3}V^3 - W + I = 0$ (V-nullcline) and $V + a - bW = 0$ (W-nullcline). It can be observed that the V-nullcline is a cubic curve in the V-W plane, while the W-nullcline is simply a straight line. Assuming the input I is fixed, then the location of the fixed point depends on the parameters a and b, which control the slope and shift of the W-nullcline. The temporal evolution of V and W is characterized by the location of the system's fixed point.

Results and Discussion

In Figure 1, we plot the phase plots obtained from solving the F-N model with zero stimulus (i.e., $I = 0$) and a time scale $\tau = 10$, while varying the values of a and b. We can see that depending on their values, the V-W curve can either hit a fixed point and the evolution ends, or exhibit a cyclic behavior. We note that for all cases, the starting point is $(V(0), W(0)) = (0, 0)$, denoted with a circle.

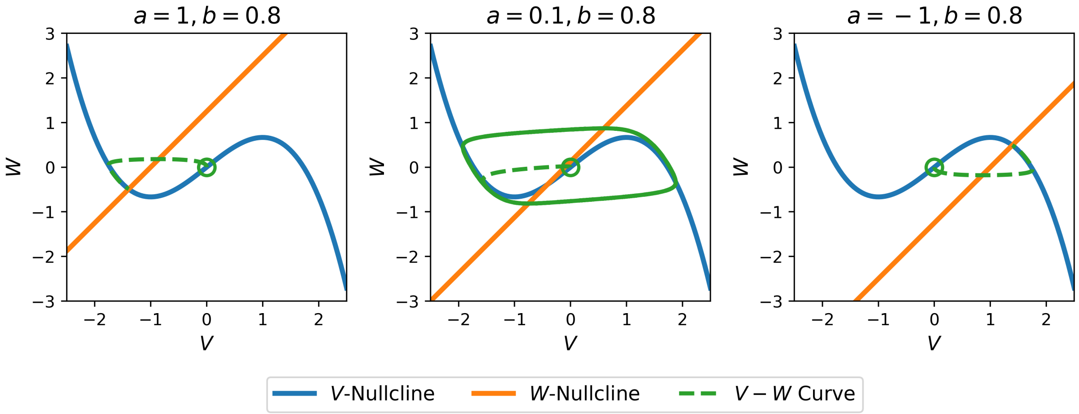

Figure 1: Plot of the V-W phase plot using different values of a and b to show the influence of model parameters on bifurcation behavior.

It is clear that the specification of the model parameters is crucial to the accurate calculation of its evolution and the prediction of possible neuron spiking activity. Moreover, for a time-dependent input current (I), the fixed point locations can vary with time, changing the neuronal activity. Hence, we propose the adoption of a forward sensitivity approach [1, 2], which combines streaming measurements with an analysis of the sensitivity of model's dynamic with respect to its components to estimate optimal values for the unknown parameters.

References

[1] S. Lakshmivarahan, J. M. Lewis, Forward sensitivity approach to dynamic data assimilation, Advances in Meteorology 2010 (2010) 1–12.

[2] S. Lakshmivarahan, J. M. Lewis, R. Jabrzemski, Forecast error correction using dynamic data assimilation, Springer, Switzerland, 2017.