## Dynamic response features and motion regimes of multimodal systems with Coulomb damping

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**Abstract**. The behaviour of single-degree-of-freedom (SDOF) systems with Coulomb friction under harmonic excitation has been extensively investigated, however little investigation has been conducted on multi-degree-of-freedom (MDOF) systems. In this contribution, Den Hartog's results for SDOF systems are extended to the MDOF case, including continuous multimodal systems, by considering a superposition of the modal behaviour. This procedure allows the analytical formulation of: (i) displacement transmissibilities and phase angles when the system response is continuous; (ii) the conditions for which continuous or stick-slip regimes occur in the response. These results are validated with numerical analyses showing an excellent agreement.

## Introduction

The development of a fundamental understanding of the friction damping effects in structural dynamics is one of the main challenges for exploring reliable mechanical and civil designs. In fact, frictional joints are found in several engineering structures, such as the roots of gas-turbine blades. In the design stage, the dynamic performance of these structures is often investigated by introducing simplified structural models and analysing their response to a harmonic excitation in the presence of Coulomb damping. Many authors have explored the dynamic behaviour of Coulomb damped SDOF systems under harmonic excitation [1, 2, 3], while the same problem is mostly unexplored when MDOF case is considered. The purpose of this contribution is the analytical and numerical investigation of the dynamic properties of: (1) discrete MDOF systems with a Coulomb contact; (2) built-up structures where a multimodal continuous system is connected to a friction damper.

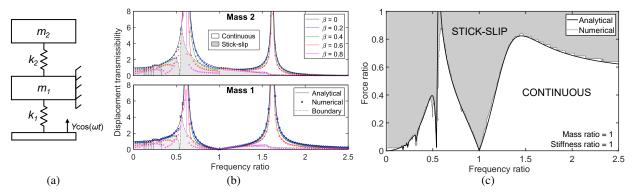


Figure 1: 2DOF system with a fixed Coulomb contact under harmonic excitation: scheme (**a**), motion regimes in the parameter space (**b**), displacement transmissibilities for the two masses for varying friction ratio  $\beta$  (**c**).

## **Results and discussions**

In reference [1], Den Hartog proposes an analytical approach for the determination of the response amplitude and phase angle of a SDOF system with Coulomb friction to a harmonic excitation, under the assumption that such a response is continuous. In the same contribution, also a condition for the presence of stops in the motion is described. Despite the nonlinearity of the friction force, Den Hartog investigates the steady-state response during time intervals where the problem is linear. Such intervals can also be defined if a MDOF system is considered, provided that only a single friction contact occurs. Under these conditions, a standard modal analysis procedure can be introduced for applying Den Hartog's approach to each vibrating mode of the system and, therefore, considering a superposition of the modal behaviour. The proposed approach allows to obtain analytical formulations for: (i) the displacement transmissibilities when the response is continuous, as shown in Fig.1b for the case of 2DOF system with Coulomb damping (Fig.1a); (ii) the boundaries between continuous and stick-slip regimes, which can be observed in Fig.1b in the form of lower bounds for the analytical transmissibilities and are further represented in Fig.1c in a 2-D parameters space. The procedure is also extended to the analysis of a friction damper, modelled a SDOF system, connected to a multimodal continuous system to evaluate analytical expressions of transmissibilities and bounds.

## References

- [1] Den Hartog J. P. (1930) Forced vibrations with combined viscous and Coulomb damping. Trans Am Soc Mech Eng 53:107-115.
- [2] Hong H., Liu C. (2000) Non-sticking oscillation formulae for Coulomb friction under harmonic loading. J S Vib 229:1171-1192.
- [3] Marino, L., Cicirello, A., Hills, D.A. (2019) Displacement transmissibility of a Coulomb friction oscillator subject to joined base-wall motion. *Nonlinear Dyn* 98:2595-2612.