Slow-fast dynamics of an oversteer vehicle

Alois Steindl*, Johannes Edelmann* and Manfred Plöchl*
* *Institute for Mechanics and Mechatronics, TU Wien, Vienna, Austria*

Abstract. We investigate the dynamics of a two-wheel oversteering vehicle with a modified brush model for the tyre forces after loss of stability by a Hopf bifurcation from a steady-state cornering motion. By including the dynamics of the vehicle's forward speed, we observed a supercritical Hopf bifurcation from a steady state cornering motion showing the Canard phenomenon and large relaxation oscillations, which are typical for singularly perturbed systems.
In this presentation we extend the previously used brush model for the tyre forces yielding a constant friction coefficient for large slip velocities by considering a decrease of the friction coefficient, after a saturation value has occured.

Introduction

The basic planar two-wheel vehicle model with rear-wheel drive has been chosen to study the motion and stability properties of an automobile ([1]).

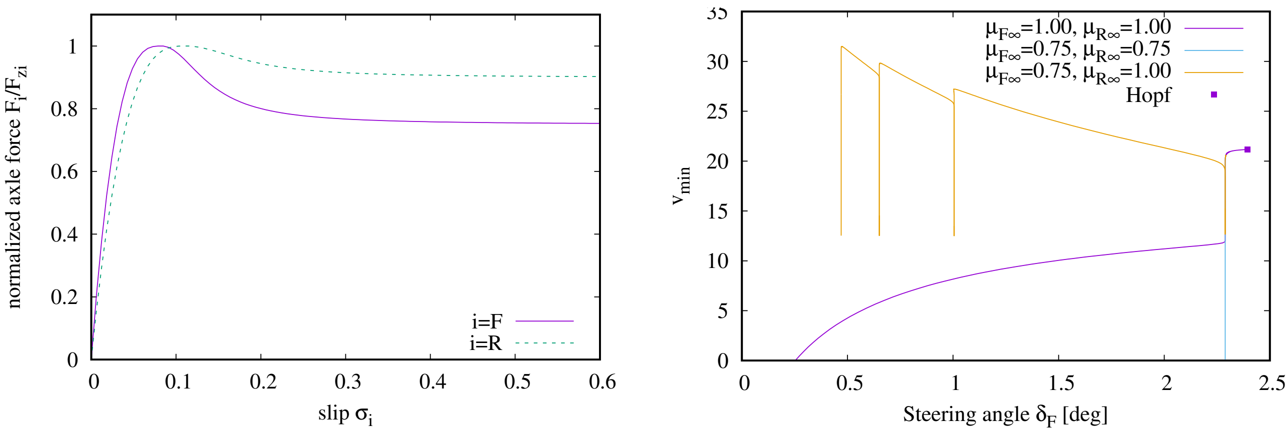

Figure 1: Normalized slip characteristics of front and rear simplified tyre/axle model with $\mu_{F\infty} = 0.75$ and $\mu_{R\infty} = 0.9$ (left). Bifurcation diagram for varying steering angles, fixed drive moment and different tyre characteristics (right).

For the tyre characteristics we use a modified version of the brush model used in [1], which mimics the behaviour of the "magic curve" given in [2] for $\sigma_i > \sigma_{\mathrm{sl},i}$,

$$F_i = \mu_i F_{zi} f(\theta_i \sigma_i), \quad \text{with } f(\sigma) = \begin{cases} 3\sigma - 3\sigma^2 + \sigma^3 & \text{for } \sigma \le 1 \\ \mu_{i\infty} + (1 - \mu_{i\infty})/(1 + 2(\sigma - 1)^2) & \text{for } \sigma > 1 \end{cases} \tag{1}$$

where F_i with $i \in \{F,\, R\}$ represents the magnitude of the total front and rear tyre/axle force, respectively.

Results and discussion

By varying the control parameters δ_F (steering angle) and M_r (drive moment) and searching the corresponding stationary motions a Hopf bifurcation is detected and we obtain a supercritical family of periodic solutions, which are shown in Fig. 1. At a small amplitude of the oscillation we observe a "canard explosion" ([3]): While the parameter δ_F remains approximately constant, the amplitude increases significantly.
When σ_i increases beyond the sliding limit $\sigma_{\mathrm{sl},i}$, the curves start to differ:

- For $\mu_{F\infty} = \mu_{R\infty} = 1$ the canard explosion stops and relaxation oscillations occur, with δ_F decreasing again. This branch ends, when the velocity $v(t)$ vanishes for some t.
- For $\mu_{F\infty} = \mu_{R\infty} = 0.75$ the relaxation oscillations vanish and the canard explosion extend down to $v_{\mathrm{min}} = 0$ with almost constant δ_F.

  The same behaviour is obtained for $(\mu_{F\infty}, \mu_{R\infty}) = (1, 0.75)$.
- For $(\mu_{F\infty}, \mu_{R\infty}) = (0.75, 1)$ we find a cascade of canard explosions at different values of δ_F, with regular branches in-between.

References

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[3] Krupa, M., Szmolyan, P.: Relaxation Oscillation and Canard Explosion. Journal of Differential Equations 174, 312–368 (2001)