

# On Learning the Impact Dynamics of a Physical Beam Structure Coupled to a Multi-Stable Continuum

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**Abstract.** In this study we are characterizing the physical dynamical processes developed in the domain of an actual complex structure formed by a viscoelastic natural continuum on which a natural multi-stable flexible continuum is attached by joints with pin bearings. Being a quite complicated structural system, a natural machine learning approach is followed where sensory data clouds, simultaneous distributed acceleration measurements, over the whole complex structure are formed and then naturally, mathematically-and-geometrically, are reduced by advanced proper orthogonal decomposition transforms into their intrinsic characteristic directions (computation-and-identification). The reduction analysis of the data cloud-it allows mechanics to exhibit in geometric terms-reveals the multi-scale slow-fast structure of the principal acceleration signals as well as their distribution over the points of measurement. The dynamical processes induced by an impact over the multi-stable continuum are contained within its domain exhibiting ultra-fast decay as a result of strong bending-stretching-shearing interactions.

## Introduction

It has been discovered that nonlinear mechanical structural systems with coexisting multiple static equilibria may admit dynamical processes with irreversible energy flow [1,2]. From the standpoint of designing structural-machinery systems with nonlinearities, it is a challenge to find a way to explore-discover-characterize whether embedding of multiple-stable flexible continua in a core flexible continuum opens up a systematic way to design-discover advanced flexible mechanisms for energy manipulation. The physical continuum level is attractive given the sensor-and-machine learning potentiality. This challenge of discovering-and-building this interesting class of complex mechanical systems can be potentially met by a physical model approach. Figure 1 presents a continuum modified by embedding within it a multi-stable flexible continuum to trigger interactions between the slow invariant manifolds and fast time scales: they need the continuum to manifest themselves. Slow and fast invariant manifolds separate time scales but also can make them interact [3]. The physical dynamics are sampled by a set of sensors detecting the transversal acceleration field in local time-and-space varying coordinates. The advanced proper orthogonal decomposition (POD) transform reduces the data cloud, the dataset as geometric object, of the typical motion into essential low-dimensional modal dynamics (principal signals). This is the essential information and contains the time and space scales activated in the continuum of complicated geometry and joints.

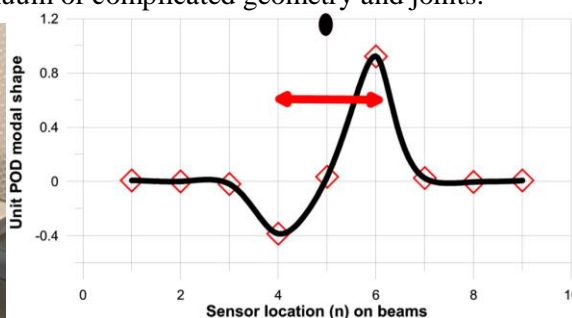


Figure 1: Photographic view of a lab set-up of a coupled two-beam system, with at least three equilibrium states, instrumented with eight accelerometers, five in the core beam and three in the attached continuum, to sample the transversal acceleration field. Right figure: the space distribution, POD modal shape, of the 1<sup>st</sup> principal acceleration signal is localized over the domain of the multiple-equilibria continuum (red thick line).

Free wave-vibration dynamical processes were excited systematically by impacting the core and the attached multi-stable substructure. The whole structure data clouds were processed and the critical reduced dynamics were computed. A remarkable result, among others, is to learn that for an impact-induced motion at the domain of the multi-stable continuum, the vibration energy remains, despite wave propagation, in this flexible continuum with a very small amount leaking to the core continuum, the cantilevered steel beam. In the sequel, it is dissipated in a coupled free vibration process involving nonlinear interaction between bending-stretching-shearing slow motions and a multitude of fast vibrations [1].

## References

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