Pendulum under bi-harmonic excitation

Shagen Shaginyan*, **Margarita Kovaleva*** and Leonid Manevitch*
*N.N.Semenov Institute of Chemical Physics, Russian academy of Sciences, Moscow, Russia

Abstract. We present a study of a pendulum under bi-harmonic forcing near primary resonance. The earlier developed semi-inverse method for the study of essentially nonlinear systems is applied which allowed us to separate "slow" and "fast" dynamics of the system. An analysis carried out in the study has revealed conditions for excitation of relaxation oscillations exhibited by a weakly damped system. The results of analytical approximations and numerical simulations are observed to be in a good agreement.

Introduction

In our work we considered oscillations of pendulum under external forcing in form of two harmonic forces with weakly detuned frequencies. The system can demonstrate the plethora of the dynamical regimes, some of them will be the subject of the future work. We will concentrate on the moderate forcing and interesting weakly modulated and strongly modulated dynamics, including the self-sustained regimes. We do not restrict our study only with low-amplitude excitations, taking into account increasing effect of the non-linearity at higher amplitudes. In such a system we consider the single resonance manifold approximation using a semi-inverse procedure to obtain a reduced-order system[1]. The technique allows to analyse the stationary states and dynamic transitions. To investigate the phase space transitions we use the Limiting Phase Trajectory (LPT) technique[1], which allows to predict different transitions of the phase space. The similar work was made by Starosvetsky [2] for the weakly non-linear system under biharmonic forcing. However, our paper deals with a more complex system as the nonlinearity cannot be supposed weak in the selected regimes of motion.

System under consideration and main results

The considered system consists of pendulum which is subject to external forcing with two frequencies being close to each other and also being close the primary resonance. We do not suppose amplitudes of the excitation being small.

$$q''(t) + \sin[q(t)] + 2\epsilon\gamma q'(t) = 2\epsilon(f_1 \sin(\omega_1 t) + f_2 \sin(\omega_2 t))$$

where ε is a small parameter, f_1 and f_2 are the amplitudes of external forcing with frequencies ω_1 and ω_2, correspondingly. Such a system can demonstrate regimes presented at the Fig.1. Supposing that the motion of the pendulum happens close to the primary resonance with some yet undefined frequency ω, which is not defined yet, we rewrite the equation as follows[1]:

$$q''(t) + \omega^2 q - \varepsilon\mu(\omega^2 q + \sin[q(t)]) + 2\epsilon\gamma q'(t)$$
$$= 2\epsilon f_1\sqrt{1+\zeta^2+2\zeta\cos[(\omega_2-\omega_1)t]}\,\sin\left(\frac{\omega_1+\omega_2}{2}t + \arctan\left[\frac{1-\zeta}{1+\zeta}\tan\left(\frac{\omega_2-\omega_1}{2}t\right)\right]\right)$$

where μ is a bookkeeping parameter, $\zeta = \frac{f_2}{f_1}$, and $\omega_2 - \omega_1 = \xi\epsilon^2$.

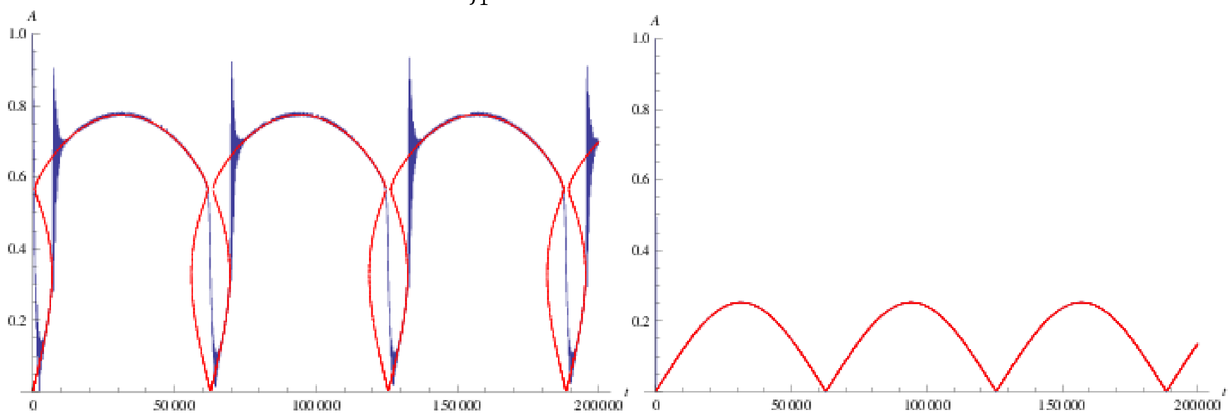

Figure 1: Evolution of the initial system (blue line) compared with the analytical solutions (red line).

Semi-inverse method together with the multiple-scale procedure allow to define different SIM branches which demonstrate the evolution in the super-slow time scale. Under the condition of low-damping, relaxation oscillations correspond to the transition from upper branch of the SIM to the lower one and vice versa. Realization of such regime strong depends on the bifurcations of the SIM branches.

References

[1] Kovaleva M., Smirnov V., and Manevitch L. (2019) Nonstationary dynamics of the sine lattice consisting of three pendula (trimer). *Phys. Rev. E* **99**: 012209.

[2] Starosvetsky Y. and Manevitch, L. I. (2011) Nonstationary regimes in a Duffing oscillator subject to biharmonic forcing near a primary resonance. *Phys. Rev. E* **83**: 046211.