## Analytic Methods for Estimating the Effects of Stochastic Intermittent Loading on Fatigue Crack Nucleation

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**Abstract**. The Palmgren-Miner rule, the industry standard for measuring the accumulation of material fatigue damage, neglects memory effects and time-ordering dependence, and therefor runs into difficulties dealing with highly intermittent loads with long tailed distributions. In this presentation, we introduce a fast semi-analytical solution for intermittent loads based on the Serebrinsky-Ortiz model of material fatigue, which greatly improves integration speed while still conservatively identifying early failures. Further, we develop a framework for estimating the distribution of failure times for stochastic intermittent loads which reproduces the long left tail.

## Introduction

The modern energy industry increasingly relies on enormously capital intensive structures, which are placed in extreme conditions and subject to extreme loads that vary throughout the structure's expected lifetime. Minimizing lifetime costs require safe-life engineering and a conservative assessment of failure probabilities. Unfortunately, nondestructive measurement of fatigue is both difficult and expensive [7].

For many classes of structure, fatigue loads have important intermittent character [6]. Alternative approaches to avoid the time cost of Monte Carlo simulations include statistical linearization [1], hierarchical modeling [8], structured sampling methods [4], and optimal experimental design [2].



Figure 1: a,b) Schematic representation of the SO evolution of material stiffness. c) Comparison of analytic failure time distribution with fast and full Monte Carlo simulations, as well as Miner's Rule. d) Comparison of failure times distribution for three different intermittent load distributions.

## **Results and Discussion**

In this presentation, we develop a fast analytical algorithm for integrating the fatigue effect of intermittent stochastic loads based on the Serebrinsky-Ortiz model [5]. This method is based on a domain decomposition approach [3], where most (small) load cycles contribute to accumulated fatigue damage by a load-spectrum dependent 'median fatigue increment', while large load spikes are handled separately.

Our algorithm performs significantly faster than the full Serebrinsky-Ortiz integration, without introducing substantial errors. For materials subject to intermittent loads, our algorithm demonstrates both greater variance in cycles-to-failure than the Palmgren-Miner rule, as well as a heavy left tail of early failures that corresponds to large load spikes at encountered at particularly sensitive times.

In addition, we develop an analytical method for calculating the probability density of failure times for stochas-

tic loads, which is is motivated by our observation that at most two extreme loads contribute meaningfully to early material failure. Comparison of the probability densities computed via simulation and analytic method show that the direct analytic method can match the long left tail without expensive Monte Carlo simulations.

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