Modified non-linear Schrödinger models, $\mathcal{CPT}$ symmetry, dark solitons and infinite towers of anomalous charges

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Abstract. We summarize the formulation of the problem and motivations to study the quasi-integrable deformations of the soliton models, such as the non-linear Schrödinger model. It is also presented a summary of the previous works on this subject. We also describe the main results and findings for the modified (defocusing) non-linear Schrödinger models.

Introduction

Some non-linear field theory models with important physical applications and solitary wave solutions are not integrable. Recently, some deformations of integrable models, such as sine-Gordon and Korteweg-de Vries [1, 2], which exhibit soliton-type properties, have been put forward. Quasi-integrability properties of the deformations of the integrable models have recently been examined in the frameworks of the anomalous zero-curvature formulations [3] and the deformations of the Riccati-type pseudo-potential approach [1, 2, 4]. In [4] it has been considered the properties of the modified (focusing) non-linear Schrödinger model with bright solitons.

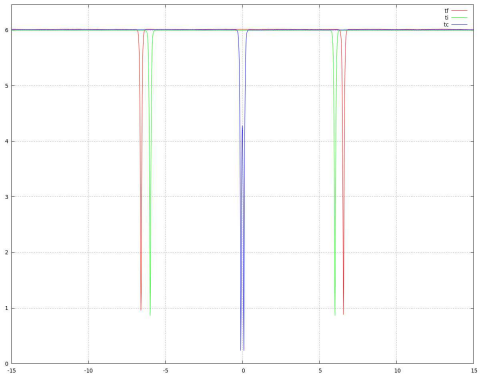

Figure 1: Reflection of two dark solitons of the cubic-quintic NLS model (1) is plotted for $\epsilon = -0.01$, $|\psi_0| = 6$, $\eta = 2.5$. The initial solitons (t_i =green line) travel in opposite direction with velocity $v \approx 1.97\sqrt{2}$. They partially overlap (t_c= blue line) in their closest approximation and then reflect to each other. The dark solitons after collision are plotted as a red line (t_f).

Modified (defocusing) non-linear Schrödinger models and dark solitons

Quasi-integrability properties of the modified non-linear (defocusing) Schrödinger models (dMNLS) of type $i\partial_t\psi(x,t) + \partial_x^2\psi(x,t) - [\frac{\delta V(|\psi|^2)}{\delta|\psi|^2}]\psi(x,t) = 0$, where $\psi \in C$ and $V : R_+ \to R$ is the deformed potential, are considered. The models exhibit infinite towers of infinitely many anomalous charges; i.e. charges satisfying quasi-conservation laws which give rise to asymptotically conserved charges. For the standard (defocusing) NLS this property holds for N-dark soliton solutions which are invariant under a charge conjugation, special shifted parity and delayed time reversal symmetry $\mathcal{CP}_s\mathcal{T}_d$. We compute numerically some anomalous charges of a particular dMNLS model given by the non-integrable cubic-quintic NLS model

$$i\frac{\partial\psi(x,t)}{\partial t} + \frac{\partial^2\psi(x,t)}{\partial x^2} - \left(2\eta|\psi(x,t)|^2 - \frac{\epsilon}{2}|\psi(x,t)|^4\right)\psi(x,t) = 0, \quad \eta > 0, \ \epsilon \in R \tag{1}$$

Our numerical simulations show the elastic scattering of dark solitons (see Fig. 1) for a wide range of values of the set $\{\eta, \epsilon\}$ and a variety of amplitudes, velocities and relative initial phases. Since the modified NLS equations are quite ubiquitous, our results may find potential applications in several areas of non-linear science.

References

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