# Amplitude-Voltage Response of Parametric Resonance Electrostatically Actuated DWCNT Resonators

**Dumitru I. Caruntu***, Ezequiel Juarez*

* *Department of Mechanical Engineering, University of Texas Rio Grande Valley, Edinburg, TX, USA*

**Abstract**. Reduced Order Models (ROM) with up to five modes of vibration (terms) are used to investigate the amplitude-voltage of parametric resonance of coaxial vibrations of electrostatically actuated double-walled carbon nanotubes (DWCNTs). ROM with one term is solved using the Method of Multiple Scales (MMS), and all other ROMs are solved using a continuation and bifurcation method AUTO-07P, and also numerical integration. Intertube van der Waals, electrostatic, and damping are forces acting on the DWCNT. Soft AC excitation and small viscous damping forces are assumed. Euler-Bernoulli beam assumptions are considered for the DWCNT. MMS and AUTO solutions show an excellent agreement in low amplitudes. The importance of the results in this paper is the effect of AC frequency on the voltage response of the DWCNT.

## Introduction

Sumio Iijima discovered the fullerene-based carbon nanotube (CNT) [1]. DWCNTs are specific type of CNTs comprised of two concentric carbon nanotubes with one tube nested within the other. DWCNTs may be used in applications like lasers [2], sensors [3], and transistors and switches [4]. The actuating mechanism in the application of DWCNTs as resonator sensors for mass sensing is “parallel plate” electrical actuation. Pull-in instability is a phenomenon that occurs when investigating nonlinear behavior under electrostatic actuation [5]. The 0th order solution for free response of DWCNTs has been previously investigated [6].

## Amplitude-voltage response

The governing equations of motion of the two carbon-nanotubes (CNTs) are given by [6], Fig. 1 (Left):

$$\rho A_1 \frac{\partial^2 y_1}{\partial t^2} + EI_1 \frac{\partial^4 y_1}{\partial x^4} = f_{vdWT-T}\,;\;\; \rho A_2 \frac{\partial^2 y_2}{\partial t^2} + EI_2 \frac{\partial^4 y_2}{\partial x^4} = -b\frac{\partial y_2}{\partial t} - f_{vdWT-T} + f_{elec}\,;\;\; f_{vdWT-T} = C_1(y_2 - y_1) + C_3(y_2 - y_1)^3 \quad (1)$$

where $y_1, y_2$ are CNTs deflections, $A_1, A_2$ cross-sectional areas, $I_1, I_2$ cross-sectional moments of inertia, of inner and outer tubes, respectively, $x$ is axial longitudinal coordinate, $t$ time, $\rho$ density, $E$ Young’s modulus, and $b$ damping coefficient. Also $f_{vdWT-T}$ is the intertube van der Waals force where $C_1, C_3$ are van der Waals interlayer interaction coefficients. The electrostatic force is given by an AC voltage $V = V_0 \cos\Omega t$. $V_0$ and $\Omega$ are the amplitude and circular frequency of the AC voltage, respectively. Figure 1 (Right) shows the effect of frequency on the amplitude-voltage response on DWCNT.

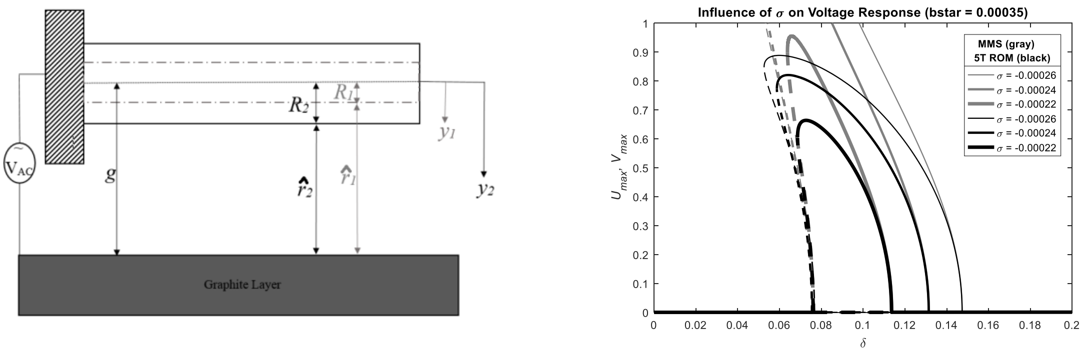


Figure 1: (Left) Electrostatically actuated DWCNT cantilever resonator. (Right) Effect of frequency on the amplitude-voltage response of parametric resonance of coaxial vibrations of electrostatically actuated DWCNT.

For numerical simulations a typical DWCNT with a length of 200 nm, inner tube radius 0.35 nm, outer tube radius 0.7 nm, gap $g$ of 50 nm, has been considered. Dimensionless deflections $U_{max}$ and $V_{max}$ with respect to the gap of the tips of the two CNTs are equal since coaxial vibrations are investigated. The amplitude-voltage response consists of subcritical and supercritical bifurcations, with bifurcation points of zero amplitude, and a bifurcation of non-zero amplitude. As the detuning frequency $\sigma$ increases the supercritical bifurcation point is shifted to lower voltage values and the amplitude of non-zero amplitude bifurcation point reduces significantly, while the subcritical bifurcation point is not affected.

## References

[1] Monthioux M., Kuznetsov V. L. (2006), *CARBON* 44.
[2] Santiago E. V., et al. (2016), *Opt. Laser Technol.* 84, pp. 53–58.
[3] Li Y. et al. (2014), *Sensors Actuators, B Chem*., 203: 63–70.
[4] Wang S. et al. (2007) *Carbon* N.Y. 45(4): 760–765.
[5] Dequesnes M. et al. (2002), *Nanotechnology*, 13: 120-131.
[6] Caruntu D., Juarez E., (2019) *Nonlinear Dynamics*, 98(4): 3095-3112.