A New Numerical Inverse Wavelet Transform and its Application to Dynamics

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Abstract. Being able to extract temporal and spatial scales in both stationary and nonstationary signals is of great importance, especially due to their applications in mechanical system identification, identification of effective scales in fluid flows, etc. In this research, we propose a harmonic decomposition approach based on inverse continuous wavelet transform that overcomes the limitation of its predecessors, such as mode-mixing issues, and also is capable of extracting stationary, non-stationary and closely spaced harmonics. Among the application of this approach is linear modal analysis which results in revelation of the natural frequencies, modal damping coefficients and modal matrices of linear system of oscillators. Furthermore, this method is capable of quantifying how a nonlinear spring can cause a single degree-of-freedom oscillator to redistribute energy in frequency-domain.

Introduction

In studying dynamical systems, it is often crucial to identify and study the different time-scales that govern their behaviors. This is particularly of interest for system identification purposes. System identification techniques typically rely on frequency, e.g. Fourier transform, or time-frequency, e.g. Short-Time Fourier Transforms and Wavelet Transforms, analysis. One of the more widely used of these time-frequency analysis techniques is the Empirical Mode Decomposition (EMD) [1]. In this research we employ inverse wavelet transform for harmonic decomposition in a signal. Due to its mathematical form, wavelet transform is both easier and more accurate to implement [2], hence, by manipulating the original formula describing inverse continuous wavelet transform, we bring it into form suitable for fast and efficient harmonic decomposition of stationary and non-stationary oscillating components.

Results and discussion

In this approach, the user selects harmonic regions of interest in the wavelet transform of a signal, $y(t)$, and then applies the modified inverse wavelet transform formula to each selected harmonic region. We show how this approach for harmonic decomposition overcomes the mode-mixing problem of its predecessor through analyzing an artificial signal, $y(t) = y_1(t) + y_2(t) = \sin 3t + \cos[60(1 - e^{-0.05t}) + 5t]$. The wavelet transform of $y(t)$ is shown in figure 1a and the excellent comparisons between the extracted components, i.e., $\tilde{y}_1(t)$ and $\tilde{y}_2(t)$, and the exact signals, $y_1(t)$ and $y_2(t)$, are shown figures 1b and 1c, respectively.

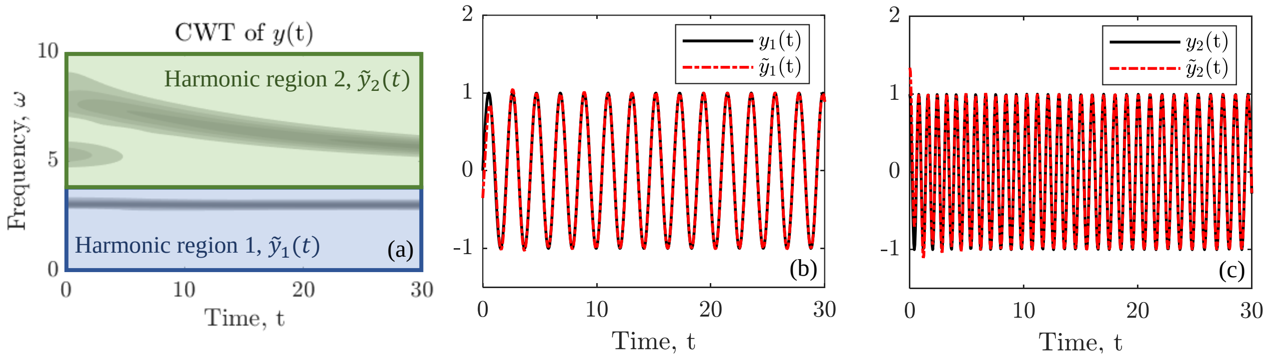

Figure1: (a) Wavelet transform of $y(t)$, with harmonic regions 1 and 2, corresponding to $\tilde{y}_1(t)$ and $\tilde{y}_2(t)$, respectively. (b) comparison between the extracted harmonic, $\tilde{y}_1(t)$, and the exact signal component, $y_1(t)$. (c) comparison between the extracted harmonic, $\tilde{y}_2(t)$, and the exact signal component, $y_2(t)$.

Furthermore, we demonstrate that it can be implemented for linear modal analysis, revealing natural frequencies, modal damping coefficients and modal matrices. Lastly, we show that by implementing this method for harmonic decomposition, we can study and quantify how a nonlinear stiffness in a nonlinear dynamical system can scatter energy in frequency domain through generation of high-frequency harmonics. Studying a certain type of geometric nonlinearity described in [7], we explain how the geometric parameter affects energy distribution among the generated harmonics.

References

[1] Huang, N.E., et al. (1971), *The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis.* Proceedings of the Royal Society of London. Series A: mathematical, physical and engineering sciences. **454**:903-995.

[2] Addison, P.S. (2017) The Illustrated Wavelet Transform Handbook: Introductory Theory and Applications in Science, Engineering, Medicine and Finance, Second Edition. CRC Press.