Enhancing network synchronization in limit cycle oscillators via dynamic coupling

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Abstract. In certain networks of limit cycles oscillators, in which the interaction between the nodes is via static feedback, the interval of coupling strength values for which the network exhibits stable synchronization may shrink as long as the number of oscillators in the network is increased. In this paper, we show by means of an example, that this limitation may be relaxed or even eliminated by replacing every static interconnection by a dynamic one.

Introduction

It is well-known that a network of dynamical systems may show synchronized or collective behavior [1]. However, depending on the network topology, and/or on the individual node dynamics, the 'synchronizability' properties of some networks may be considerably affected when more nodes are added, see, e.g. [1, 2].

Problem statement

Consider a network of relaxation oscillators described by [3]:

$$\dot{\boldsymbol{x}}_i = \boldsymbol{f}(\boldsymbol{x}_i) - k\boldsymbol{B}\sum_{j=1}^{N} G_{ij}y_j, \quad y_i = \boldsymbol{C}\boldsymbol{x}_i, \quad \boldsymbol{f}(\boldsymbol{x}_i) = \left[\begin{array}{c} \frac{1}{\mu}\left(x_{1i} - \frac{1}{3}x_{1i}^3 - x_{2i}\right) \\ x_{1i} + \mu x_{2i} \end{array}\right], \quad i = 1, 2, ...N, \tag{1}$$

where $\boldsymbol{x}_i \in \mathbb{R}^2$ is the state vector corresponding to node i, function $\boldsymbol{f} : \mathbb{R}^2 \to \mathbb{R}^2$ describes the isolated node dynamics, $\boldsymbol{B} \in \mathbb{R}^2$ and $\boldsymbol{C} \in \mathbb{R}^{1\times 2}$ are the input and output vectors, respectively, y_i is the output, G_{ij} determines the strength of interaction between nodes i and j and is determined by the network topology, N is the number of nodes, and k is the overall coupling strength. If for example, we consider a star topology and the vectors $\boldsymbol{B} = [1\ 0]^T$ and $\boldsymbol{C} = [1\ 0]$, then we have a 'dramatic' situation: *the interval of coupling strength values, for which the synchronous solution $\boldsymbol{x}_1(t) = \boldsymbol{x}_2(t) = \boldsymbol{x}_N(t)$ is stable, compresses down toward the origin as the number of nodes in the network is increased.* This is shown in Figure 1a). How to enhance the network synchronizability in such case? Is it possible? These questions are addressed in this paper.

Proposed network synchronization scheme with dynamic couplings

Here, we propose the following network synchronization scheme:

$$\dot{\boldsymbol{x}}_i = \boldsymbol{f}(\boldsymbol{x}_i) + \boldsymbol{B}_1\boldsymbol{h}_i, \quad i = 1, 2, \ldots, N, \tag{2}$$

$$\dot{\boldsymbol{h}}_i = \boldsymbol{A}\boldsymbol{h}_i - k\sum_{j=1}^{N} G_{ij}\boldsymbol{B}_2\boldsymbol{x}_j, \qquad \boldsymbol{A} = \left[\begin{array}{cc} -\alpha & 1 \\ -\gamma_1 & -\gamma_2 \end{array}\right], \tag{3}$$

where $\boldsymbol{x}_i$ and $\boldsymbol{f}$ are as given in (1), $\boldsymbol{h}^i \in \mathbb{R}^2$ is the state vector of the dynamic coupling, $\boldsymbol{B}_1 \in \mathbb{R}^{n\times 2}$, and $\boldsymbol{B}_2 \in \mathbb{R}^{2\times n}$ are coupling vectors and $\boldsymbol{A} \in \mathbb{R}^{2\times 2}$ is a matrix containing the positive "control parameters" α, γ_1, and γ_2 of the dynamic coupling. If we consider again the star network example discussed above, then by properly tuning the parameters in matrix $\boldsymbol{A}$ by using the master stability function approach, it is possible to enlarge the synchronization region, as shown in Figures 1b) and 1c). Furthermore, when synchronization is achieved, i.e. when $\boldsymbol{x}_1(t) = \boldsymbol{x}_2(t) = \boldsymbol{x}_N(t)$, then the coupling variables $\boldsymbol{h}_i$ vanish asymptotically.

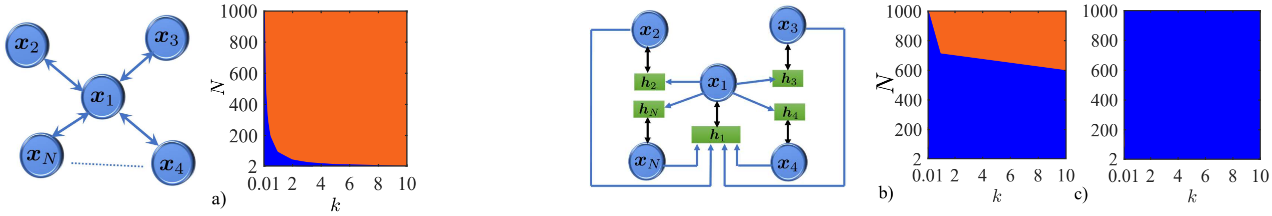

Figure 1: Stability regions in the $(k, N)-$plane computed using the master stability function approach. In all panels, blue: stable synchronization, i.e. the largest transverse Lyapunov exponent is negative , red: unsynchronized behavior, i.e. the largest transverse Lyapunov exponent is positive. a) Classical network given in Eq. (1) with static couplings. b) Proposed network given in (2,3) with $\alpha = 1, \gamma_1 = 1, \gamma_2 = 6k$. c) Proposed network given in (2,3) with $\alpha = 1, \gamma_1 = 1, \gamma_2 = 12k$.

Conclusions

Dynamic coupling enlarges the synchronization region in networks of limit cycle oscillators.

References

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