Nonlinear Modal Analysis through the generalization of the eigenvalue problem: Applications for Dissipative Dynamics

Nidish Narayanaa Balaji and Matthew R. W. Brake

Department of Mechanical Engineering, William Marsh Rice University, Houston, TX, USA

Abstract. The present work considers the possibility of generalizing the eigenvalue problem for non-linear dynamical systems in a manner that is similar to the study of linear dynamics. Previous efforts have indicated that the linear eigenvalue problem, occurring in the study of linear undamped structures, can be generalized using a generalization of the concept of constrained minimization of Rayleigh Quotients. The current investigation considers a similar extension for the quadratic eigenvalue problem which occurs in the study of damped linear dynamics. Special emphasis is placed on structures with nonlinear dissipative elements.

Introduction

The concept of Nonlinear Modal Analysis (NMA) has become a very popular analytical as well as computational tool in the study of non-linear structural systems. There have been several formulations of a computational procedure for this [1], including time-domain methods based on a shooting-type approach and frequency domain methods based on equivalent undamped systems. Quasi-static approaches have also gained popularity in recent years, and the present work considers a formulation that is similar to such approaches [2].

Recent efforts by the authors [2] have indicated that a generalization of the constrained minimization of Rayleigh quotients provides an interesting modal analysis approach (termed Rayleigh Quotient-based NMA (RQNMA)) for non-linear structures that is closely related to the other methods. This approach was, however, lacking in that even linear damping must be considered only in a “decoupled” fashion. Non-linear damping can, on the other hand, be considered only after a “marching” procedure (rather expensive, computationally) is conducted to characterize the complete hysteretic characteristics. The present study, drawing inspiration from the analysis of damped linear systems using quadratic eigenvalue problems (see [3], for instance), considers the possibility of a more generalized formulation of the quadratic eigenvalue problem that can overcome both of the above limitations. Additionally, synthesis of forced responses from such studies are explored in multi-degree-of-freedom systems considering the co-existence of multiple non-linear modes in a response simultaneously.

Results and Discussions

For a dynamical system with inertial, damping, and stiffness matrices denoted by $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$; the degree-of-freedom vector denoted by $\utilde{u}$; nonlinear and external forces denoted by $\utilde{f}_{nl}(\utilde{u},\dots)$ and $\utilde{f}_{ex}(t)$; and time derivatives denoted by a “dot” $\dot{(\cdot)}$ (with equations of motion written as $\mathbf{M}\ddot{\utilde{u}} + \mathbf{C}\dot{\utilde{u}} + \mathbf{K}\utilde{u} + \utilde{f}_{nl}(\utilde{u}) = \utilde{f}_{ex}(t)$), the algebraic equations that must be solved for RQNMA for a given modal amplitude q are,

$$\begin{aligned} \mathbf{K}\utilde{u} + \utilde{f}_{nl}(\utilde{u}) - \lambda\mathbf{M}\utilde{u} &= 0,\\ \utilde{u}^T\mathbf{M}\utilde{u} - q^2 &= 0. \end{aligned} \tag{1}$$

The multiplier, λ in the above (also an unknown), is interpreted as the square of the nonlinear resonant frequency, by analogy with linear dynamics. Note here that the damping matrix does not occur in this formulation and has to be considered in a decoupled fashion by estimating the modal damping ratio as $\utilde{u}^T\mathbf{C}\utilde{c}/(2\sqrt{\lambda}$ (through similar analogy). Dissipative nonlinearities, however, can not be faithfully characterized using this approach alone (see [2] for details).

For damped dynamics, a generalization of the quadratic eigenvalue problem (see [3] for detailed discussions on damped linear dynamics) of the form,

$$\begin{aligned} \left[\mathbf{M}s^2 + \mathbf{C}s + \mathbf{K}\right]\bar{\utilde{u}} + \bar{\utilde{\mathcal{F}}}_{nl}(\utilde{u}) &= 0\\ \utilde{u}^T\left[2s\mathbf{M} + \mathbf{C}\right]\utilde{s} - 2sq^2 &= 0, \end{aligned} \tag{2}$$

is sought. Note that the nonlinear term here is represented using a different symbol since the generalization of all types of nonlinearities is not very trivial and is considered in detail in the paper. A crucial difference from the previous formulation is that this system of equations is posed in the complex domain.

References

[1] Kerschen, G., Peeters, M., Golinval, J. C., Vakakis, A. F. (2009) Nonlinear normal modes, Part I: A useful framework for the structural dynamicist. *Mechanical systems and signal processing* **23(1)**:170-194.

[2] Balaji, N. N., Brake, M. R. (2020) A quasi-static non-linear modal analysis procedure extending Rayleigh quotient stationarity for non-conservative dynamical systems. *Computers & Structures* **230**:106184.

[3] Adhikari, S. (2013) Structural dynamic analysis with generalized damping models: analysis. John Wiley & Sons.