Vibration analysis of a multi-DOF impact oscillator with multiple linear constraints

Wei Dai and Jian Yang

Faculty of Science and Engineering, University of Nottingham Ningbo China, Ningbo, China

Abstract. This paper investigates the nonlinear dynamic behaviour of a 3-degree-of-freedom (3DOF) impact oscillator system with multiple constraints. The system model comprises a 3DOF chain oscillators and three extra springs as linear constraints for oscillating masses. The steady-state responses of the system subjected to a harmonic excitation force are obtained by the harmonic balance(HB) approximations and a numerical integrations. The effects of the spring stiffness of the linear constraints on the dynamic responses of the masses are studied. The results show that the implementation of the linear constraints may have nonlinear hardening effect on the frequency responses of the system. It can lead to multiple solutions and bifurcations near the resonance frequencies of different vibration modes. Moreover, super-harmonic response components are observed and an extra resonance peak may be obtained near corresponding frequency of the second vibration mode.

Introduction

The impact oscillator model is widely used for studying the vibro-impact phenomena within many of engineering dynamic systems such as robot joints [1]. The inclusion of rigid or elastic constraint in a system can bring in non-smooth stiffness nonlinearities, resulting in rich nonlinear behaviour in the dynamic response including bifurcations and chaotic motions [2]. The elastic constraint may also alter the vibration transmission characteristics between sub-structures of the integrated system [3]. As shown in Fig. 1, three linear single-DOF oscillators are coupled into a chain oscillator, the motion of which is restrained by three constraints. The static equilibrium positions of the three masses, where $x_1 = x_2 = x_3 = 0$ and the springs k_1, k_2, k_3, k_4 are un-stretched, are set as a reference. Three linear constraints C1, C2 and C3 with identical stiffness k_s shown in Fig. 1 are for the mass m_1, m_2 and m_3, respectively. When the springs in three constraints are un-stretched and the masses are in equilibrium, i.e., $x_1 = x_2 = x_3 = 0$, the left-hand-side terminals of the three linear constraints are placed at a distance of d to the right of masses m_1, m_2 and m_3, respectively.

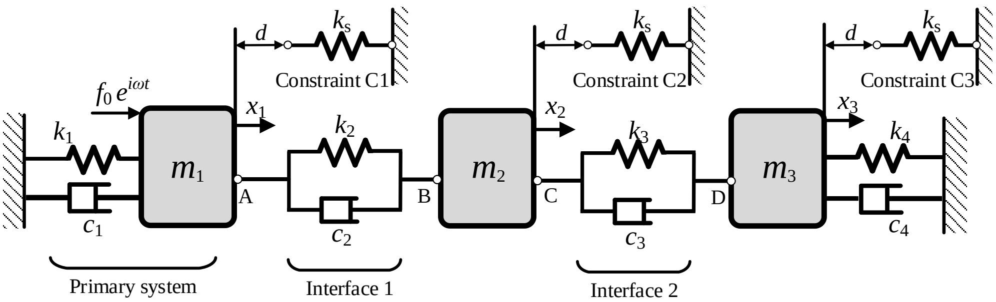

Figure 1: Schematic of the 3DOF impact oscillator system with multiple constraints.

The equations of the motion of the system in a matrix form are

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 & 0 \\ -c_2 & c_2 + c_3 & -c_3 \\ 0 & -c_3 & c_4 + c_3 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_4 + k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} + \begin{Bmatrix} f_{C1} \\ f_{C2} \\ f_{C3} \end{Bmatrix} = \begin{Bmatrix} f_0 e^{i\omega t} \\ 0 \\ 0 \end{Bmatrix}, \tag{1}$$

where f_{C1}, f_{C2} and f_{C3} represent the forces applied by the constraints C1, C2 and C3 to the mass m_1, m_2 and m_3, respectively. These equations are then solved by HB approximation and the numerical integrations. The steady-state frequency responses and vibration transmission levels within the system are determined.

Results and discussions

The results show that the inclusion of the linear constraints can lead to rich nonlinear phenomena including super-harmonic responses and multiple solution branches. The resonance peaks in the curves of the steady-state displacement response of the masses are bending to the high-frequencies and an extra peak may appear in the frequency response curve of the secondary mass m_2. Multiple solutions and bifurcations can be found near the resonance peaks of the responses. The influence of the multiple constraints on the vibration transmission is examined using force transmissibility and vibration power flow as performance indices.

References

[1] Yin S., Ji J., Wen G. (2019) Complex Near-grazing Dynamics in Impact Oscillators. *Int J Mech Sci* **156**:106-122

[2] Nordmark A.B. (1991) Non-periodic Motion Caused by Grazing Incidence in an Impact Oscillator, *J Sound Vib* **145**:279-297.

[3] Dai W., Yang J., Shi B. (2020) Vibration Transmission and Power Flow in Impact Oscillators with Linear and Nonlinear Constraints *Int J Mech Sci* **168**:105234.