

# Effect of mechanical nonlinearity on dynamics of MEMS gyroscope drive mode oscillator circuit with automatic gain control and phase locked loop

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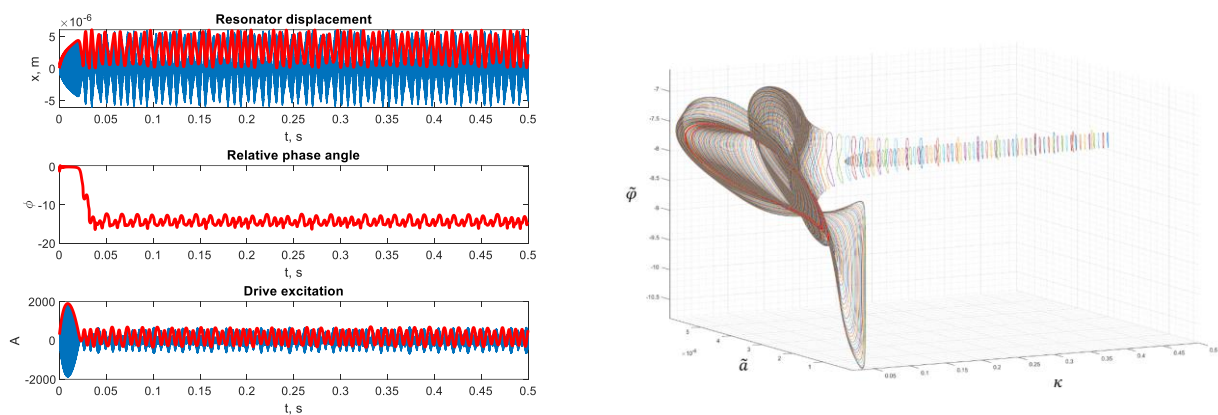
**Abstract.** This work presents the results of a qualitative study of the dynamics of the micromechanical resonator considering the action of the control system with PLL and AGC loops. Particular attention is directed to the study of features associated with the presence of a nonlinear component of the elastic restoring force of the resonator. It was found that the choice of control system parameters based on the stability analysis of the working resonant mode for a mechanically linear model, in general case, does not provide the required stabilization of the amplitude and relative phase of oscillations. Numerical modeling revealed attractive (stable) complex multi-frequency modes of oscillations with variable amplitude and phase. An analytical study of the mechanisms of the appearance of these modes and their evolution is performed when changing key system parameters: the degree of mechanical nonlinearity, the Q factor of the resonator and the frequency of the poles of the low-pass filter of the PLL and AGC loops.

## Introduction

In this work, we study the dynamics of a micromechanical resonator under the action of a control system designed for resonant frequency tuning and stabilization of the amplitude of oscillations, considering effect of nonlinearity of the resonator restoring force. The frequency setting of the electrostatic motor for resonance is provided by a phase locked loop (PLL) [1]. The stabilization of the amplitude of the primary oscillations is achieved by the operation of the automatic gain control (AGC) circuit [2]. As far as the authors know, a qualitative (parametric) study of the general properties of dynamical processes in systems of this type has not previously been encountered in the literature.

## Results and discussion

For the considered design cases, the real areas of stable operation of the control system were determined (which, as was found, do not coincide with the stability areas of the working stationary mode due to the presence of hidden attractors in the phase space of the system [3]). The figure 1, a shows the results of numerical integration of the initial and averaged systems describing the dynamics of the resonator for post-critical value of the dimensionless nonlinearity parameter  $\kappa$ .



a) Dynamics of the resonator at  $Q = 100, \kappa = 2.5\%$

b) Evolution of the limit cycle with a change in  $\kappa$

Figure 1

A technique of numerical continuation has been developed to determine parametric areas of stable operation of PLL-AGC control circuit. Figure 1, b shows the evolution and branching of limit cycles of MEMS resonator for slow variables  $\tilde{a}, \tilde{\varphi}$  as  $\kappa$  changes. Significant difficulties associated with a qualitative analysis of the dynamics of high-Q controlled resonators are noted. However, for this case, the models presented above give exact numerical results that are of immediate practical importance.

## References

- [1] Horowitz R. et al. Stability and Resolution Analysis of a Phase-Locked Loop Natural Frequency Tracking System for MEMS Fatigue Testing. *Journal of Dynamic Systems, Measurement and Control*. 2002. 124, 599-605
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- [3] Kuznetsov N.V. et al. Hidden attractors in dynamical models of phase-locked loop circuits: Limitations of simulation in MATLAB and SPICE. *Commun. Nonlinear Sci. Numer. Simulat.* 2017. 51, 39-49